

Identical Parallel Machine Scheduling Problems:
Structural patterns, bounding techniques and
solution procedures

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