



Universität Ulm
Institut für Stochastik

**Parametric Approximation Formulas for the Distributions
of Cost Functionals in Spatial Stochastic Networks**

Dissertation

zur Erlangung des Doktorgrades Dr. rer. nat. der Fakultät für
Mathematik und Wirtschaftswissenschaften der Universität Ulm

vorgelegt von

David Werner Neuhäuser

Juni 2015

Contents

| | | |
|----------|--|-----------|
| I | Introduction and Mathematical Foundations | 1 |
| 1 | Introduction | 3 |
| 1.1 | Aims and Motivation | 3 |
| 1.2 | Outline | 6 |
| 1.3 | Software | 7 |
| 2 | Preliminaries from Stochastic Geometry | 9 |
| 2.1 | Basic Definitions and Notation | 9 |
| 2.2 | Random Measures | 11 |
| 2.2.1 | Basic Definitions and Properties | 11 |
| 2.2.2 | Palm Distribution | 12 |
| 2.3 | Random Point Processes | 12 |
| 2.3.1 | Point Processes as Random Counting Measures | 12 |
| 2.3.2 | Basic Definitions and Properties | 13 |
| 2.3.3 | Palm Distribution and Typical Point | 14 |
| 2.3.4 | Poisson Point Processes | 15 |
| 2.3.5 | Cox Point Processes | 16 |
| 2.4 | Marked Point Processes | 18 |
| 2.4.1 | Basic Definitions and Properties | 18 |
| 2.4.2 | Palm Distribution | 19 |
| 2.4.3 | Jointly Stationary Marked Point Processes | 19 |
| 2.5 | Random Closed Sets | 20 |
| 2.6 | Random Geometric Graphs | 21 |
| 2.6.1 | Fully Connected Graphs with Convex Cells | 22 |
| 2.6.2 | Fully Connected Graphs with Non-Convex Cells | 28 |
| 3 | An Introduction to Copulas | 31 |
| 3.1 | Basic Definitions and Sklar's Theorem | 31 |
| 3.2 | Archimedean Copulas | 32 |
| 3.2.1 | Definition in Two Dimensions | 32 |
| 3.2.2 | Examples | 33 |
| 3.2.3 | Generalisation to Higher Dimensions | 35 |
| 3.3 | Elliptical Copulas | 36 |

| | | |
|---|--|-----------|
| 3.3.1 | Basic Definitions | 36 |
| 3.3.2 | Examples | 36 |
| 3.4 | Measures of Association | 37 |
| 3.5 | Calibrating Fits for Multivariate Distributions | 40 |
| II Network Model and Basic Simulation Algorithms | | 43 |
| 4 | The Stochastic Subscriber Line Model | 45 |
| 4.1 | Layer 1: The Geometry Model | 45 |
| 4.2 | Layer 2: The Network Model | 46 |
| 4.3 | Layer 3: The Topology Model | 47 |
| 4.3.1 | e-Closeness | 47 |
| 4.3.2 | g-Closeness | 48 |
| 4.4 | Typical Shortest-Path Lengths with Respect to e- and g-Closeness | 48 |
| 5 | Simulation Algorithms for the Typical Cox-Voronoi Cell | 53 |
| 5.1 | Overview of Existing Simulation Algorithms | 54 |
| 5.2 | Direct Simulation Algorithm in the Delaunay Case | 57 |
| 5.2.1 | Explicit Description of the Palm Version of a Stationary PDT | 57 |
| 5.2.2 | Acceptance-Rejection Scheme | 58 |
| 5.2.3 | Simulation Algorithm for the Typical Cox-Voronoi Cell | 59 |
| 5.3 | Simulation Algorithm in the STIT Case | 61 |
| 5.3.1 | The Typical Cox-Voronoi Cell | 62 |
| 5.3.2 | Palm Version of a STIT Tessellation | 63 |
| III Shortest-Path Trees | | 65 |
| 6 | Modelling Shortest-Path Trees – the Backbone | 67 |
| 6.1 | Extraction of Shortest-Path Trees from Typical Segment Systems | 68 |
| 6.2 | Modelling Backbones of Shortest-Path Trees by Copulas | 69 |
| 6.2.1 | Data Preprocessing and Case-by-Case Analysis | 70 |
| 6.2.2 | Fitting Parametric Marginal Distributions | 72 |
| 6.2.3 | Choosing a Suitable Copula Type | 74 |
| 6.2.4 | Same Distance-Peak as Endpoints | 74 |
| 6.2.5 | Recombination of Cases | 75 |
| 6.3 | Possible Extensions | 76 |
| 7 | Modelling Shortest-Path Trees – the Two Half-Trees | 77 |
| 7.1 | Modelling Joint Functionals of Half-Trees | 77 |
| 7.1.1 | Fitting Procedure for the Marginal Densities | 78 |
| 7.1.2 | Choosing Suitable Copulas | 81 |
| 7.2 | Distributional Results | 82 |

| | | |
|-----------|--|------------|
| 7.2.1 | Parametric Approximation Formulas for the Margins | 82 |
| 7.2.2 | Fitted Parametric Copulas | 84 |
| 7.2.3 | Infinitely Sparse and Infinitely Dense Road Systems | 84 |
| 7.3 | Model Validation | 86 |
| 7.3.1 | Visual Validation | 86 |
| 7.3.2 | Multivariate Wald-Wolfowitz Test | 91 |
| 7.4 | Possible Extensions | 92 |
| 8 | Modelling Shortest-Path Trees in Sparse Networks | 95 |
| 8.1 | Sparse Networks in Contrast to Dense Networks | 95 |
| 8.2 | Direct Simulation of the Tree in the Sparse Case | 97 |
| 8.2.1 | Random Vector Representing | 97 |
| 8.2.2 | Preprocessing Data, Scenario Analysis and Subsequent Fitting | 98 |
| 8.3 | Distributional Results | 99 |
| 8.3.1 | Conditional Distribution of the Tree in Scenario I | 99 |
| 8.3.2 | Conditional Distribution of the Tree in Scenario II | 100 |
| 8.3.3 | Conditional Distribution of the Tree in Scenario III | 102 |
| 8.4 | Model Validation | 104 |
| 8.4.1 | Visual Validation | 104 |
| 8.4.2 | Multivariate Wald-Wolfowitz Test | 105 |
| 8.5 | Possible Extensions | 107 |
| 9 | Tail Dependence in Sparse Networks | 109 |
| 9.1 | A Closer Look at Cumbersome Tails | 109 |
| 9.2 | Modelling Extremal Data | 111 |
| 9.2.1 | Preprocessing the Data | 111 |
| 9.2.2 | Fitting Parametric Marginal Distributions | 112 |
| 9.2.3 | Fitting a Parametric Copula | 113 |
| 9.2.4 | Inverting the Symmetrisation | 114 |
| 9.3 | Modelling Non-Extremal Data | 116 |
| 9.3.1 | Fitting Parametric Marginal Distributions | 116 |
| 9.3.2 | Fitting a Parametric Copula | 117 |
| 9.4 | Acceptance-Rejection Scheme | 117 |
| 9.5 | Model Validation | 118 |
| 9.5.1 | Visual Validation | 118 |
| 9.5.2 | Measures of Association | 119 |
| 9.6 | Possible Extensions | 120 |
| IV | Further Results and Extensions | 123 |
| 10 | Miscellaneous Results for Relevant Cost Functionals | 125 |
| 10.1 | Typical Shortest-Path Lengths | 125 |
| 10.1.1 | Distributional Properties | 125 |

| | | |
|-----------|--|------------|
| 10.1.2 | Limit Theorem | 127 |
| 10.1.3 | Monte Carlo Estimator | 128 |
| 10.2 | Limit Theorems for Cox-Voronoi Cells on Iterated Tessellations . . . | 130 |
| 10.2.1 | Some Auxiliary Results | 130 |
| 10.2.2 | Vanishing Initial Tessellation | 131 |
| 10.2.3 | Vanishing Component Tessellation | 133 |
| 10.3 | Numerical Results | 135 |
| 11 | Multi-Level Hierarchical Network Model | 141 |
| 11.1 | Modification of the SSLM | 141 |
| 11.2 | Distributional Results of Cost Functionals | 147 |
| 11.2.1 | Typical Shortest-Path Lengths | 147 |
| 11.2.2 | Typical Total Fibre Lengths | 151 |
| 11.3 | Limit Theorems | 152 |
| 11.3.1 | Typical Shortest-Path Lengths | 152 |
| 11.3.2 | Typical Total Fibre Lengths | 154 |
| 11.4 | Numerical Results | 155 |
| 11.4.1 | Typical Shortest-Path Lengths | 155 |
| 11.4.2 | Typical Total Fibre Lengths | 156 |
| 11.5 | Possible Extensions | 157 |
| | Bibliography | 159 |
| | Nomenclature | 167 |
| | Abstract | 171 |
| | Zusammenfassung | 173 |
| | Acknowledgement | 177 |
| | Publications and Presentations | 179 |
| | Curriculum Vitae | 181 |
| | Erklärung | 183 |