

Discrete-time Asset Pricing Models in Applied Stochastic Finance

P.C.G. Vassiliou

ISTE

 **WILEY**

Table of Contents

| | |
|--|----|
| Preface | xi |
| Chapter 1. Probability and Random Variables | 1 |
| 1.1. Introductory notes | 1 |
| 1.2. Probability space | 2 |
| 1.3. Conditional probability and independence | 8 |
| 1.4. Random variables | 12 |
| 1.4.1. Discrete random variables | 14 |
| 1.4.2. Bernoulli random variables | 15 |
| 1.4.3. Binomial random variables | 15 |
| 1.4.4. Geometric random variables | 16 |
| 1.4.5. Poisson random variables | 17 |
| 1.4.6. Continuous random variables | 18 |
| 1.4.7. Exponential random variables | 20 |
| 1.4.8. Uniform random variables | 21 |
| 1.4.9. Gamma random variables | 21 |
| 1.4.10. Normal random variables | 22 |
| 1.4.11. Lognormal random variables | 23 |
| 1.4.12. Weibull random variables | 23 |
| 1.5. Expectation and variance of a random variable | 24 |
| 1.6. Jointly distributed random variables | 28 |
| 1.6.1. Joint probability distribution of functions of random variables | 30 |
| 1.7. Moment generating functions | 32 |
| 1.8. Probability inequalities and limit theorems | 37 |
| 1.9. Multivariate normal distribution | 44 |
| Chapter 2. An Introduction to Financial Instruments and Derivatives | 49 |
| 2.1. Introduction | 49 |
| 2.2. Bonds and basic interest rates | 50 |

| | |
|---|-----------|
| 2.2.1. Simple interest rates | 51 |
| 2.2.2. Discretely compounded interest rates | 51 |
| 2.2.3. Continuously compounded interest rate | 52 |
| 2.2.4. Money-market account | 53 |
| 2.2.5. Basic interest rates | 55 |
| 2.2.5.1. Treasury rate | 55 |
| 2.2.5.2. LIBOR rates | 55 |
| 2.2.6. Time value of money | 55 |
| 2.2.7. Coupon-bearing bonds and yield-to-maturity | 56 |
| 2.3. Forward contracts | 58 |
| 2.3.1. Arbitrage | 59 |
| 2.4. Futures contracts | 60 |
| 2.5. Swaps | 60 |
| 2.6. Options | 62 |
| 2.6.1. European call option | 62 |
| 2.6.2. European put option | 63 |
| 2.6.3. American call option | 63 |
| 2.6.4. American put option | 64 |
| 2.6.5. Basic problems and assumptions | 65 |
| 2.7. Types of market participants | 67 |
| 2.7.1. Hedgers | 67 |
| 2.7.2. Speculators | 67 |
| 2.7.3. Arbitrageurs | 67 |
| 2.8. Arbitrage relationships between call and put options | 67 |
| 2.9. Exercises | 69 |
| Chapter 3. Conditional Expectation and Markov Chains | 71 |
| 3.1. Introduction | 71 |
| 3.2. Conditional expectation: the discrete case | 72 |
| 3.3. Applications of conditional expectations | 75 |
| 3.3.1. Expectation of the sum of a random number of random variables | 76 |
| 3.3.2. Expected value of a random number of Bernoulli trials with probability of success being a random variable | 77 |
| 3.3.3. Number of Bernoulli trials until there are k consecutive successes | 78 |
| 3.3.4. Conditional variance relationship | 79 |
| 3.3.5. Variance of the sum of a random number of random variables . . | 80 |
| 3.4. Properties of the conditional expectation | 81 |
| 3.5. Markov chains | 85 |
| 3.5.1. Probability distribution in the states of a Markov chain | 90 |
| 3.5.2. Statistical inference in Markov chains | 94 |
| 3.5.3. The strong Markov property | 97 |
| 3.5.4. Classification of states of a Markov chain | 100 |
| 3.5.5. Periodic Markov chains | 104 |

| | |
|---|------------|
| 3.5.5.1. Cyclic subclasses | 106 |
| 3.5.5.2. Algorithm for the cyclic subclasses | 109 |
| 3.5.6. Classification of states | 112 |
| 3.5.7. Asymptotic behavior of irreducible homogenous Markov chains | 115 |
| 3.5.8. The mean time of first entrance in a state of Markov chain | 126 |
| 3.5.9. The variance of the time of first visit into a state of a Markov chain | 129 |
| 3.6. Exercises | 131 |
| Chapter 4. The No-Arbitrage Binomial Pricing Model | 137 |
| 4.1. Introductory notes | 137 |
| 4.2. Binomial model | 138 |
| 4.3. Stochastic evolution of the asset prices | 141 |
| 4.4. Binomial approximation to the lognormal distribution | 143 |
| 4.5. One-period European call option | 145 |
| 4.6. Two-period European call option | 150 |
| 4.7. Multiperiod binomial model | 153 |
| 4.8. The evolution of the asset prices as a Markov chain | 154 |
| 4.9. Exercises | 158 |
| Chapter 5. Martingales | 163 |
| 5.1. Introductory notes | 163 |
| 5.2. Martingales | 164 |
| 5.3. Optional sampling theorem | 169 |
| 5.4. Submartingales, supermartingales and martingales convergence theorem | 178 |
| 5.5. Martingale transforms | 182 |
| 5.6. Uniform integrability and Doob's decomposition | 184 |
| 5.6.1. Doob decomposition | 186 |
| 5.7. The snell envelope | 187 |
| 5.8. Exercises | 190 |
| Chapter 6. Equivalent Martingale Measures, No-Arbitrage and Complete Markets | 195 |
| 6.1. Introductory notes | 195 |
| 6.2. Equivalent martingale measure and the Randon-Nikodým derivative process | 196 |
| 6.3. Finite general markets | 204 |
| 6.3.1. Uniqueness of arbitrage price | 210 |
| 6.3.2. Equivalent martingale measures | 213 |
| 6.4. Fundamental theorem of asset pricing | 215 |
| 6.5. Complete markets and martingale representation | 222 |

| | |
|---|------------|
| 6.6. Finding the equivalent martingale measure | 228 |
| 6.6.1. Exploring the vital equations and conditions | 234 |
| 6.6.2. Equivalent martingale measures for general finite markets | 237 |
| 6.7. Exercises | 238 |
| Chapter 7. American Derivative Securities | 241 |
| 7.1. Introductory notes | 241 |
| 7.2. A three-period American put option | 242 |
| 7.3. Hedging strategy for an American put option | 249 |
| 7.4. The algorithm of the American put option | 254 |
| 7.4.1. Algorithm of the American put option | 254 |
| 7.4.1.1. Pricing of the American put option | 254 |
| 7.4.1.2. Trading strategy for hedging | 254 |
| 7.5. Optimal time for the holder to exercise | 255 |
| 7.6. American derivatives in general markets | 262 |
| 7.7. Extending the concept of self-financing strategies | 266 |
| 7.8. Exercises | 269 |
| Chapter 8. Fixed-Income Markets and Interest Rates | 273 |
| 8.1. Introductory notes | 273 |
| 8.2. The zero coupon bonds of all maturities | 274 |
| 8.3. Arbitrage-free family of bond prices | 278 |
| 8.4. Interest rate process and the term structure of bond prices | 282 |
| 8.5. The evolution of the interest rate process | 290 |
| 8.6. Binomial model with normally distributed spread of interest rates | 293 |
| 8.7. Binomial model with lognormally distributed spread of interest rates | 296 |
| 8.8. Option arbitrage pricing on zero coupon bonds | 298 |
| 8.8.1. Valuation of the European put call | 298 |
| 8.8.2. Hedging the European put option | 300 |
| 8.9. Fixed income derivatives | 302 |
| 8.9.1. Interest rate swaps | 304 |
| 8.9.2. Interest rate caps and floors | 307 |
| 8.10. T -period equivalent forward measure | 308 |
| 8.11. Futures contracts | 317 |
| 8.12. Exercises | 319 |
| Chapter 9. Credit Risk | 323 |
| 9.1. Introductory notes | 323 |
| 9.2. Credit ratings and corporate bonds | 324 |
| 9.3. Credit risk methodologies | 326 |
| 9.3.1. Structural methodologies | 326 |
| 9.3.2. Reduced-form methodologies | 327 |

| | |
|---|------------|
| 9.4. Arbitrage pricing of defaultable bonds | 327 |
| 9.5. Migration process as a Markov chain | 330 |
| 9.5.1. Change of real-world probability measure to equivalent T^* -forward measure | 331 |
| 9.6. Estimation of the real world transition probabilities | 334 |
| 9.7. Term structure of credit spread and model calibration | 337 |
| 9.8. Migration process under the real-world probability measure | 341 |
| 9.8.1. Stochastic monotonicities in default times | 344 |
| 9.8.2. Asymptotic behavior | 350 |
| 9.9. Exercises | 352 |
| Chapter 10. The Heath-Jarrow-Morton Model | 355 |
| 10.1. Introductory notes | 355 |
| 10.2. Heath-Jarrow-Morton model | 356 |
| 10.2.1. Evolution of forward rate process | 356 |
| 10.2.2. Evolution of the savings account and short-term interest rate process | 358 |
| 10.2.3. Evolution of the zero-coupon non-defaultable bond process | 359 |
| 10.2.4. Conditions on the drift and volatility parameters for non-arbitrage | 360 |
| 10.3. Hedging strategies for zero coupon bonds | 362 |
| 10.4. Exercises | 364 |
| References | 365 |
| Appendices | 374 |
| A. Appendix A | 375 |
| A.1. Introductory thoughts | 375 |
| A.2. Genesis | 376 |
| A.3. The decisive steps | 378 |
| A.4. A brief glance towards the flow of research paths | 387 |
| B. Appendix B | 391 |
| B.1. Introduction | 391 |
| B.2. The main theorem | 392 |
| Index | 395 |