

# **Analytical Modeling of Solute Transport in Groundwater**

Using Models to Understand the Effect of Natural Processes  
on Contaminant Fate and Transport

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