Bilinear $\mathscr{H}_2$-optimal Model Order Reduction with applications to thermal parametric systems

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