

# Advances in Combinatorial Optimization

**Linear Programming Formulations of the  
Traveling Salesman and Other Hard  
Combinatorial Optimization Problems**

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