

# **MACHINE ANALYSIS WITH COMPUTER APPLICATIONS FOR MECHANICAL ENGINEERS**

**James Doane**

*Frontier-Kemper Constructors, Indiana, USA*

**WILEY**

# Contents

<b>Preface</b>	<b>xv</b>
<b>Acknowledgments</b>	<b>xvii</b>
<b>About the companion website</b>	<b>xix</b>
<b>1</b>	<b>1</b>
<b>1.1</b>	<b>1</b>
<b>1.1.1</b>	<b>1</b>
<i>Brief History of Machines</i>	1
<b>1.1.2</b>	<b>5</b>
<i>Why Study Machine Analysis?</i>	5
<b>1.1.3</b>	<b>6</b>
<i>Differences between Machine Analysis and Machine Design</i>	6
<b>1.2</b>	<b>6</b>
<b>1.2.1</b>	<b>6</b>
<i>Importance of Units</i>	6
<b>1.2.2</b>	<b>6</b>
<i>Unit Systems</i>	6
<b>1.2.3</b>	<b>8</b>
<i>Units of Angular Motion</i>	8
<b>1.2.4</b>	<b>8</b>
<i>Force and Mass</i>	8
<b>1.3</b>	<b>10</b>
<b>1.3.1</b>	<b>10</b>
<i>Machine versus Mechanism</i>	10
<b>1.3.2</b>	<b>10</b>
<i>Simple Machines</i>	10
<b>1.3.3</b>	<b>11</b>
<i>Static Machine Analysis</i>	11
<b>1.3.4</b>	<b>14</b>
<i>Other Types of Machines</i>	14
<b>1.4</b>	<b>14</b>
<b>1.4.1</b>	<b>14</b>
<i>Introduction to Linkage Mechanisms</i>	14
<b>1.4.2</b>	<b>14</b>
<i>Types of Links</i>	14
<b>1.4.3</b>	<b>15</b>
<i>Types of Joints</i>	15
<b>1.5</b>	<b>16</b>
<b>1.5.1</b>	<b>17</b>
<i>Crank–Rocker Mechanisms</i>	17
<b>1.5.2</b>	<b>17</b>
<i>Slider–Crank Mechanisms</i>	17
<b>1.5.3</b>	<b>18</b>
<i>Toggle Mechanisms</i>	18
<b>1.5.4</b>	<b>19</b>
<i>Quick Return Mechanisms</i>	19
<b>1.5.5</b>	<b>19</b>
<i>Straight Line Mechanisms</i>	19
<b>1.5.6</b>	<b>20</b>
<i>Scotch Yoke Mechanism</i>	20
<b>1.6</b>	<b>21</b>
<b>1.6.1</b>	<b>21</b>
<i>Introduction to Gears</i>	21
<b>1.6.2</b>	<b>22</b>
<i>Spur Gears</i>	22
<b>1.6.3</b>	<b>23</b>
<i>Helical Gears</i>	23

1.6.4	<i>Bevel Gears</i>	24
1.6.5	<i>Worm Gears</i>	26
1.7	Cams	27
1.7.1	<i>Introduction to Cams</i>	27
1.7.2	<i>Disk Cams</i>	27
1.7.3	<i>Cylindrical Cams</i>	28
1.8	Solution Methods	28
1.8.1	<i>Graphical Techniques</i>	29
1.8.2	<i>Analytical Methods</i>	29
1.8.3	<i>Computer Solutions</i>	29
1.9	Methods of Problem Solving	30
1.9.1	<i>Step 1: Carefully Read the Problem Statement</i>	30
1.9.2	<i>Step 2: Plan Your Solution</i>	30
1.9.3	<i>Step 3: Solve the Problem</i>	30
1.9.4	<i>Step 4: Read the Problem Statement Again</i>	30
1.10	Review and Summary	31
	Problems	31
	Further Reading	33
<b>2</b>	<b>Essential Kinematics Concepts</b>	<b>34</b>
2.1	Introduction	34
2.2	Basic Concepts of Velocity and Acceleration	35
2.3	Translational Motion	35
2.4	Rotation about a Fixed Axis	36
2.4.1	<i>Velocity</i>	36
2.4.2	<i>Acceleration</i>	38
2.5	General Plane Motion	41
2.5.1	<i>Introduction</i>	41
2.5.2	<i>Velocity Difference and Relative Velocity</i>	41
2.5.3	<i>Relative Acceleration</i>	47
2.5.4	<i>Instant Center of Rotation</i>	51
2.6	Computer Methods	53
2.6.1	<i>Numerical Differentiation</i>	53
2.6.2	<i>Illustrative Example</i>	54
2.7	Review and Summary	58
	Problems	58
	Further Reading	65
<b>3</b>	<b>Linkage Position Analysis</b>	<b>66</b>
3.1	Introduction	66
3.2	Mobility	67
3.2.1	<i>Rigid Body Degrees of Freedom</i>	67
3.2.2	<i>Joint Mobility</i>	67
3.2.3	<i>Determining Mobility of a Planar Linkage Mechanism</i>	69
3.3	Inversion	72
3.4	Grashof's Criterion	72
3.4.1	<i>Introduction</i>	72

3.4.2	<i>Grashof Linkage</i>	73
3.4.3	<i>Non-Grashof Linkage</i>	73
3.4.4	<i>Special Case Grashof Linkage</i>	73
3.5	Coupler Curves	74
3.5.1	<i>Basic Concepts of Coupler Curves</i>	74
3.5.2	<i>Double Points</i>	74
3.5.3	<i>Hrones and Nelson Atlas</i>	75
3.6	Cognate Linkages	76
3.6.1	<i>The Roberts–Chebyshev Theorem</i>	76
3.6.2	<i>Steps for Determining Cognates</i>	76
3.6.3	<i>Cognates for Binary Link</i>	78
3.6.4	<i>Cognates for Slider–Crank Mechanism</i>	79
3.7	Transmission Angle	79
3.8	Geometrical Method of Position Analysis	80
3.8.1	<i>Essential Mathematics</i>	80
3.8.2	<i>Common Approaches for Four-Bar Mechanisms</i>	80
3.9	Analytical Position Analysis	92
3.9.1	<i>Loop Closure Equation</i>	92
3.9.2	<i>Complex Number Notation</i>	98
3.10	Toggle Positions	100
3.11	Computer Methods for Position Analysis	100
3.11.1	<i>Position Analysis Using Spreadsheets</i>	100
3.11.2	<i>Distance Formula to Solve for Output Angle</i>	101
3.11.3	<i>Computer Solutions Using MATLAB® and MathCAD</i>	102
3.12	Review and Summary	103
	Problems	103
	Further Reading	107
<b>4</b>	<b>Linkage Velocity and Acceleration Analysis</b>	<b>108</b>
4.1	Introduction	108
4.2	Finite Displacement: Approximate Velocity Analysis	109
4.3	Instantaneous Centers of Rotation	111
4.3.1	<i>Number of Instant Centers</i>	111
4.3.2	<i>Primary Instant Centers</i>	112
4.3.3	<i>Kennedy–Aronhold Theorem</i>	112
4.3.4	<i>Locating Instant Centers for Typical Four-Bar Mechanisms</i>	112
4.3.5	<i>Locating Instant Centers for Slider–Crank Mechanisms</i>	114
4.3.6	<i>Locating Instant Centers for Other Mechanisms</i>	114
4.3.7	<i>Velocity Analysis Using Instant Centers</i>	115
4.4	Graphical Velocity Analysis	119
4.4.1	<i>Basic Concepts</i>	119
4.4.2	<i>Component Method</i>	122
4.4.3	<i>Parameter Studies</i>	124
4.5	Analytical Velocity Analysis Methods	125
4.5.1	<i>Introduction</i>	125
4.5.2	<i>Vector Method</i>	126
4.5.3	<i>Loop-Closure Method</i>	127
4.5.4	<i>Differentiation of Position Coordinate Equation</i>	129

4.6	Graphical Acceleration Analysis Methods	130
4.7	Analytical Acceleration Analysis Methods	134
4.8	Kinematic Analysis of Linkage Mechanisms with Moving Slides	135
	4.8.1 <i>Sliding Motion</i>	135
	4.8.2 <i>Motion Relative to Rotating Axes</i>	138
	4.8.3 <i>Coriolis Component</i>	141
	4.8.4 <i>Geneva Mechanisms</i>	144
4.9	Review and Summary	147
	Problems	147
	Further Reading	153
<b>5</b>	<b>Linkage Synthesis</b>	<b>154</b>
5.1	Introduction	154
	5.1.1 <i>Synthesis Classifications</i>	154
	5.1.2 <i>Essential Engineering Geometry and Drafting</i>	155
5.2	Synthesis	155
	5.2.1 <i>Type Synthesis</i>	155
	5.2.2 <i>Number Synthesis</i>	156
	5.2.3 <i>Dimensional Synthesis</i>	156
5.3	Two-Position Graphical Dimensional Synthesis	156
	5.3.1 <i>Introduction</i>	156
	5.3.2 <i>Using Rocker Motion from a Double Rocker Mechanism</i>	157
	5.3.3 <i>Using Rocker Motion from a Crank–Rocker Mechanism</i>	158
	5.3.4 <i>Using Coupler Motion</i>	160
	5.3.5 <i>Adding a Driver Dyad</i>	161
5.4	Three-Position Graphical Dimensional Synthesis	162
	5.4.1 <i>Using Coupler Motion</i>	162
	5.4.2 <i>Fixed Pivot Point Locations Defined</i>	163
5.5	Approximate Dwell Linkage Mechanisms	167
5.6	Quick Return Mechanisms	169
	5.6.1 <i>Time Ratio</i>	169
	5.6.2 <i>Quick Return Crank–Rocker Mechanism</i>	170
	5.6.3 <i>Whitworth Mechanism</i>	172
	5.6.4 <i>Offset Slider–Crank Mechanism</i>	174
5.7	Function Generation	176
	5.7.1 <i>Introduction</i>	176
	5.7.2 <i>Accuracy Points</i>	176
	5.7.3 <i>Chebyshev Spacing</i>	177
	5.7.4 <i>Freudenstein’s Equation: Four-Bar Linkage</i>	178
	5.7.5 <i>Error Analysis for Function Generation</i>	182
5.8	Review and Summary	182
	Problems	182
	Further Reading	189
<b>6</b>	<b>Computational Methods for Linkage Mechanism Kinematics</b>	<b>190</b>
6.1	Introduction	190
6.2	Matrix Review	190
	6.2.1 <i>Introduction</i>	190

6.2.2	<i>Matrix Notation</i>	191
6.2.3	<i>Matrix Operations</i>	191
6.2.4	<i>Representing Simultaneous Equations in Matrix Form</i>	193
6.3	Position Equations	196
6.3.1	<i>Introduction</i>	196
6.3.2	<i>Iterative Solution Method</i>	196
6.3.3	<i>MATLAB® Program Module for Calculating <math>\theta_3</math> and <math>\theta_4</math></i>	201
6.3.4	<i>Position Analysis Using MathCAD</i>	202
6.3.5	<i>Linkage Center of Mass Locations</i>	205
6.4	Velocity Analysis	206
6.4.1	<i>Numerical Differentiation</i>	206
6.4.2	<i>Derivatives of Data Containing Errors or Noise</i>	206
6.4.3	<i>Velocity Analysis in MathCAD</i>	207
6.5	Acceleration Equations	209
6.5.1	<i>Numerical Differentiation</i>	209
6.5.2	<i>Acceleration Analysis in MathCAD</i>	209
6.6	Dynamic Simulation Using Autodesk Inventor	210
6.6.1	<i>Basic Concepts</i>	210
6.6.2	<i>Kinematic Constraints</i>	211
6.6.3	<i>Kinematic Analysis Example</i>	211
6.7	Review and Summary	211
	Problems	212
	Further Reading	214
<b>7</b>	<b>Gear Analysis</b>	<b>215</b>
7.1	Introduction	215
7.2	Involute Curves	216
7.2.1	<i>Conjugate Profiles</i>	216
7.2.2	<i>Properties of Involute Curves</i>	217
7.3	Terminology	219
7.3.1	<i>Pitch Circle and Pressure Angle</i>	219
7.3.2	<i>Base Circle</i>	220
7.3.3	<i>General Gear Tooth Terminology</i>	221
7.3.4	<i>Clearance and Backlash</i>	227
7.4	Tooth Contact	228
7.4.1	<i>Involute Gear Tooth</i>	228
7.4.2	<i>Path of Contact</i>	229
7.4.3	<i>Contact Ratio</i>	231
7.4.4	<i>Interference</i>	233
7.5	Analysis of Spur Gears	234
7.5.1	<i>Basic Concepts of Spur Gears</i>	234
7.5.2	<i>Speed Ratio of Spur Gears</i>	235
7.5.3	<i>Efficiency of Spur Gears</i>	238
7.6	Analysis of Parallel Helical Gears	239
7.6.1	<i>Parallel versus Crossed Helical Gears</i>	239
7.6.2	<i>Basic Concepts of Helical Gears</i>	240
7.6.3	<i>Terminology Specific to Helical Gears</i>	240

7.6.4	<i>Efficiency of Helical Gears</i>	241
7.7	Analysis of Crossed Helical Gears	242
7.7.1	<i>Graphical Solution for a Shaft Angle of 90°</i>	242
7.7.2	<i>Graphical Solution for Other Shaft Angles</i>	244
7.7.3	<i>Versatility of Helical Gears for Nonparallel Shafts</i>	245
7.8	Analysis of Bevel Gears	246
7.8.1	<i>Basic Concepts of Bevel Gears</i>	246
7.8.2	<i>Terminology Specific to Bevel Gears</i>	247
7.8.3	<i>Speed Ratio and Direction of Rotation</i>	247
7.8.4	<i>Other Types of Bevel Gears</i>	248
7.9	Analysis of Worm Gearing	249
7.9.1	<i>Basic Concepts of Worm Gearing</i>	249
7.9.2	<i>Terminology Specific to Worm Gearing</i>	249
7.9.3	<i>Speed Ratios of Worm Gearing</i>	251
7.9.4	<i>Efficiency of Worm Gearing</i>	251
7.9.5	<i>Self-locking Condition</i>	252
7.10	Review and Summary	252
	Problems	252
	Further Reading	254
<b>8</b>	<b>Gear Trains</b>	<b>255</b>
8.1	Introduction	255
8.2	Simple Gear Trains	256
8.3	Compound Gear Trains	258
8.3.1	<i>Speed Ratio Calculations</i>	258
8.3.2	<i>Design of Compound Gear Trains</i>	260
8.4	Reverted Compound Gear Trains	262
8.5	Gear Trains with Different Types of Gears	264
8.6	Planetary Gear Trains	266
8.6.1	<i>Investigation of a Historical Application</i>	266
8.6.2	<i>Basic Planetary Gear Train</i>	267
8.6.3	<i>Speed Ratio of Planetary Gear Trains</i>	268
8.7	Differentials	273
8.8	Computer Methods for Gear Train Design	274
8.9	Review and Summary	274
	Problems	275
	Further Reading	279
<b>9</b>	<b>Cams</b>	<b>280</b>
9.1	Introduction	280
9.2	Types of Cams and Followers	281
9.2.1	<i>Common Cam Configurations</i>	281
9.2.2	<i>Follower Types</i>	281
9.3	Basic Concepts of Cam Geometry and Cam Profiles	283
9.3.1	<i>Follower Displacement</i>	283
9.3.2	<i>SVAJ Diagrams</i>	283
9.3.3	<i>General Rules of Cam Design</i>	285

9.4	Common Cam Functions	285
9.4.1	<i>Introduction to Cam Functions</i>	285
9.4.2	<i>Simple Harmonic Function</i>	286
9.4.3	<i>The Cycloidal Function</i>	287
9.4.4	<i>The 3-4-5 Polynomial Function</i>	290
9.4.5	<i>The 4-5-6-7 Polynomial Function</i>	290
9.4.6	<i>The Double Harmonic Function</i>	291
9.4.7	<i>Comparison of Cam Functions</i>	292
9.5	Using Cam Functions for Specific Applications	295
9.6	Application of Cam Functions for Double-Dwell Mechanisms	299
9.7	Application of Cam Functions for Single-Dwell Mechanisms	301
9.8	Application of Cam Functions for Critical Path Motion	308
9.8.1	<i>Basic Concept of Critical Path Motion</i>	308
9.8.2	<i>Cam Functions for Constant Acceleration</i>	308
9.8.3	<i>Cam Functions for Constant Velocity</i>	308
9.9	Cam Geometry	310
9.9.1	<i>Basic Concepts</i>	310
9.9.2	<i>Base Circle</i>	310
9.9.3	<i>Pressure Angle</i>	311
9.10	Determining Cam Size	312
9.10.1	<i>General Ideas</i>	312
9.10.2	<i>Maximum Pressure Angle for Simple Harmonic Functions</i>	314
9.10.3	<i>Maximum Pressure Angle for More Complex Cam Functions</i>	315
9.11	Design of Cam Profiles	316
9.11.1	<i>Graphical Methods for Plate Cams with In-Line Followers</i>	316
9.11.2	<i>Graphical Methods for Offset Followers</i>	320
9.12	Computer Methods for Cam Design	322
9.13	Review and Summary	322
	Problems	323
	Reference	327
<b>10</b>	<b>Vibration Theory</b>	<b>328</b>
10.1	Introduction	328
10.2	System Components	329
10.3	Frequency and Period	333
10.4	Undamped Systems	333
10.4.1	<i>Equations of Motion</i>	333
10.4.2	<i>Graphical Representation of Initial Conditions</i>	339
10.4.3	<i>Energy Methods</i>	340
10.5	Torsional Systems	344
10.6	Damped Systems	346
10.6.1	<i>Equations of Motion</i>	347
10.6.2	<i>Critically Damped Systems</i>	348
10.6.3	<i>Overdamped Systems</i>	348
10.6.4	<i>Underdamped Systems</i>	350
10.7	Logarithmic Decrement	353
10.8	Forced Vibration: Harmonic Forcing Functions	356



10.8.1	<i>Harmonic versus Periodic Functions</i>	357
10.8.2	<i>Equations of Motion for Harmonic Excitation</i>	357
10.8.3	<i>Resonance</i>	359
10.8.4	<i>Damped Response to Harmonic Excitation</i>	362
10.8.5	<i>Harmonic Support Motion with Viscous Damping</i>	369
10.9	Response of Undamped Systems to General Loading	372
10.9.1	<i>Constant Force</i>	372
10.9.2	<i>Ramp Load</i>	375
10.9.3	<i>Exponentially Decaying Motion</i>	377
10.9.4	<i>Combination of the Basic Forcing Functions</i>	378
10.10	Review and Summary	381
	Problems	381
	Further Reading	386
<b>11</b>	<b>Dynamic Force Analysis</b>	<b>387</b>
11.1	Introduction	387
11.2	Superposition Method of Force Analysis	388
11.2.1	<i>Introduction</i>	388
11.2.2	<i>Equivalent Offset Inertia Force</i>	391
11.2.3	<i>Superposition Method for Dynamic Force Analysis of a Four-Bar Mechanism</i>	394
11.3	Matrix Method Force Analysis	399
11.3.1	<i>General Concepts</i>	399
11.3.2	<i>Four-Bar Linkage</i>	399
11.4	Sliding Joint Forces	405
11.5	Energy Methods of Force Analysis: Method of Virtual Work	410
11.6	Force Analysis for Slider–Crank Mechanisms Using Lumped Mass	412
11.6.1	<i>Lumped Mass Assumption</i>	412
11.6.2	<i>Acceleration of the Slider</i>	413
11.7	Gear Forces	416
11.7.1	<i>Introduction</i>	416
11.7.2	<i>Spur Gears</i>	416
11.7.3	<i>Other Gear Types</i>	418
11.8	Computer Methods	418
11.9	Review and Summary	418
	Problems	419
	Further Reading	421
<b>12</b>	<b>Balancing of Machinery</b>	<b>422</b>
12.1	Introduction	422
12.2	Static Balancing	423
12.2.1	<i>Basic Concepts</i>	423
12.2.2	<i>Graphical Method for Rotor Balancing</i>	424
12.2.3	<i>Analytical Method for Rotor Balancing</i>	426
12.3	Dynamic Balancing	431
12.4	Vibration from Rotating Unbalance	437
12.5	Balancing Slider–Crank Linkage Mechanisms	439

12.5.1	<i>Introduction</i>	439
12.5.2	<i>Inertial Forces</i>	439
12.5.3	<i>Balancing Primary Forces</i>	441
12.5.4	<i>Illustrative Example of Slider–Crank Balancing</i>	442
12.5.5	<i>Lanchester Balancer</i>	445
12.6	Balancing Linkage Mechanisms	447
12.6.1	<i>Introduction</i>	447
12.6.2	<i>Global Center</i>	447
12.6.3	<i>Shaking Forces</i>	448
12.7	Flywheels	448
12.7.1	<i>Introduction</i>	448
12.7.2	<i>Speed Fluctuation</i>	449
12.7.3	<i>Flywheel Energy</i>	450
12.7.4	<i>Fluctuation of Energy</i>	451
12.7.5	<i>Flywheel Design</i>	452
12.7.6	<i>Flywheel Analysis for a Punching Press</i>	454
12.8	Measurement Devices	455
12.9	Computer Methods	458
12.9.1	<i>Balancing</i>	458
12.9.2	<i>Flywheels</i>	459
12.10	Review and Summary	459
	Problems	459
	References	464
	Further Reading	464
<b>13</b>	<b>Applications of Machine Dynamics</b>	<b>465</b>
13.1	Introduction	465
13.2	Cam Response for Simple Harmonic Functions	465
13.2.1	<i>Background</i>	465
13.2.2	<i>General Equation of Motion</i>	466
13.2.3	<i>Response for Simple Harmonic Cam Function</i>	467
13.3	General Response Using Laplace Transform Method	469
13.3.1	<i>Introduction</i>	469
13.3.2	<i>Basic Concepts of Laplace Transform</i>	470
13.3.3	<i>Step Functions</i>	471
13.3.4	<i>Transforms of Derivatives</i>	473
13.3.5	<i>Inverse Transforms</i>	473
13.3.6	<i>Vibration Analysis with Laplace Transforms</i>	477
13.4	System Response Using Numerical Methods	479
13.5	Advanced Cam Functions	482
13.5.1	<i>Introduction</i>	482
13.5.2	<i>Combination of Basic Cam Functions</i>	483
13.5.3	<i>Higher Order Polynomial Functions</i>	492
13.6	Forces Acting on the Follower	492
13.6.1	<i>Basic Concepts</i>	492
13.6.2	<i>Compression Spring Design</i>	492
13.7	Computer Applications of Cam Response	494

---

13.8	Internal Combustion Engines	494
	13.8.1 <i>Introduction</i>	494
	13.8.2 <i>Engine Force Analysis</i>	496
13.9	Common Arrangements of Multicylinder Engines	499
	13.9.1 <i>Introduction</i>	499
	13.9.2 <i>In-Line Engines</i>	501
	13.9.3 <i>Opposed Engines</i>	502
	13.9.4 <i>V Engines</i>	504
13.10	Flywheel Analysis for Internal Combustion Engines	504
13.11	Review and Summary	506
	Problems	506
	References	507
	Further Reading	507
	<b>Appendix A – Center of Mass</b>	<b>509</b>
	<b>Appendix B – Moments of Inertia</b>	<b>512</b>
	<b>Appendix C – Fourier Series</b>	<b>521</b>
	<b>Index</b>	<b>529</b>