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# Geochemical Modelling of Igneous Processes – Principles And Recipes in R Language

Bringing the Power of R to a Geochemical  
Community

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