QUANTUM ERROR CORRECTION

Edited by

DANIEL A. LIDAR
University of Southern California

TODD A. BRUN
University of Southern California

CAMBRIDGE
UNIVERSITY PRESS

Contents

List of contributors *page* xi
Prologue xv
Preface and guide to the reader xix
Acknowledgements xxi

Part I Background 1

1 **Introduction to decoherence and noise in open quantum systems** 3
Daniel A. Lidar and Todd A. Brun
1.1 Introduction 3
1.2 Brief introduction to quantum mechanics and quantum computing 4
1.3 Master equations 26
1.4 Stochastic error models 32
1.5 Conclusions 45

2 **Introduction to quantum error correction** 46
Dave Bacon
2.1 Error correction 46
2.2 From reversible classical error correction to simple quantum error correction 48
2.3 The quantum error-correcting criterion 56
2.4 The distance of a quantum error-correcting code 59
2.5 Content of the quantum error-correcting criterion and the quantum Hamming bound 59
2.6 Digitizing quantum noise 60
2.7 Classical linear codes 61
2.8 Calderbank, Shor, and Steane codes 64
2.9 Stabilizer quantum error-correcting codes 65
2.10 Conclusions 76
2.11 History and further reading 76

3 Introduction to decoherence-free subspaces and noiseless subsystems 78
Daniel A. Lidar
3.1 Introduction 78
3.2 A "classical decoherence-free subspace" 78
3.3 Collective dephasing decoherence-free subspace 79
3.4 Decoherence-free subspace defined and characterized 81
3.5 Initialization-free decoherence-free subspace 90
3.6 Noiseless subsystems 92
3.7 Initialization-free noiseless subsystems 98
3.8 Protection against additional decoherence sources 101
3.9 Conclusions 102
3.10 History and further reading 102

4 Introduction to quantum dynamical decoupling 105
Lorenza Viola
4.1 Motivation and overview 105
4.2 Warm up: bang-bang decoupling of qubit dephasing 107
4.3 Control-theoretic framework 110
4.4 Bang-bang periodic decoupling 113
4.5 The need for advanced decoupling design 119
4.6 Bounded-strength Eulerian decoupling 120

5 Introduction to quantum fault tolerance 126
Panos Aliferis
5.1 Quantum circuits and error discretization 127
5.2 Noisy quantum computers 130
5.3 Encoded quantum computation 142
5.4 Coarse-grained noise and level reduction 152
5.5 The quantum accuracy threshold 155
5.6 Assessment 157
5.7 History and further reading 158

Part II Generalized approaches to quantum error correction 161

6 Operator quantum error correction 163
David Kribs and David Poulin
6.1 Introduction 163
6.2 Equivalent conditions for OQEC 165
6.3 Stabilizer formalism for OQEC 169
6.4 Examples 172
6.5 Measuring gauge operators 175
6.6 Bounds for subsystem codes 177
6.7 Unitarily correctable codes 179

7 Entanglement-assisted quantum error-correcting codes 181
Todd A. Brun and Min-Hsiu Hsieh
7.1 Introduction 181
7.2 Constructing EAQECCs 184

7.3 Constructing EAQECCs from classical linear codes 195
7.4 Catalytic QECCs 197
7.5 Conclusions 199

8 Continuous-time quantum error correction 201
Ognyan Oreshkov
8.1 Introduction 201
8.2 CTQEC in an encoded basis 204
8.3 Quantum-jump CTQEC with weak measurements 207
8.4 Schemes with indirect feedback 213
8.5 Quantum jumps for Markovian and non-Markovian noise 218
8.6 Outlook 226

Part III Advanced quantum codes 229

9 Quantum convolutional codes 231
Mark Wilde
9.1 Introduction 231
9.2 Definition and operation of quantum convolutional codes 235
9.3 Mathematical formalism of quantum convolutional codes 238
9.4 Quantum shift-register circuits 244
9.5 Examples of quantum convolutional codes 249
9.6 Entanglement-assisted quantum convolutional codes 253
9.7 Closing remarks 260

10 Nonadditive quantum codes 261
Markus Grassl and Martin Rötteler
10.1 Introduction 261
10.2 Stabilizer codes 262
10.3 Characterization of nonadditive quantum codes 263
10.4 Construction of nonadditive QECCs 268
10.5 Quantum circuits 274
10.6 Conclusions 277

11 Iterative quantum coding systems 279
David Poulin
11.1 Introduction 279
11.2 Decoding 284
11.3 Turbo-codes 292
11.4 Sparse codes 297
11.5 Conclusion 305

12 Algebraic quantum coding theory 307
Andreas Klappenecker
12.1 Quantum stabilizer codes 307
12.2 Cyclic codes 317
12.3 Quantum BCH codes 318
12.4 Quantum MDS codes 325

13 **Optimization-based quantum error correction** 327
Andrew Fletcher
13.1 Limitation of the independent arbitrary errors model 327
13.2 Defining a QEC optimization problem 328
13.3 Maximizing average entanglement fidelity 331
13.4 Minimizing channel nonideality: the indirect method 336
13.5 Robustness to channel perturbations 338
13.6 Structured near-optimal optimization 340
13.7 Optimization for (approximate) decoherence-free subspaces 346
13.8 Conclusion 347

Part IV Advanced dynamical decoupling 349

14 **High-order dynamical decoupling** 351
Zhen-Yu Wang and Ren-Bao Liu
14.1 Introduction 351
14.2 Operator set preservation 351
14.3 Dynamical decoupling for multi-qubit systems 353
14.4 Concatenated dynamical decoupling 355
14.5 Uhrig dynamical decoupling 357
14.6 Concatenated Uhrig dynamical decoupling 365
14.7 Quadratic dynamical decoupling 366
14.8 Nested Uhrig dynamical decoupling 367
14.9 Pulses of finite amplitude 368
14.10 Time-dependent Hamiltonians 369
14.11 Randomized dynamical decoupling 372
14.12 Experimental progress 373
14.13 Discussion 374

15 **Combinatorial approaches to dynamical decoupling** 376
Martin Rötteler and Pawel Wocjan
15.1 Introduction 376
15.2 Combinatorial bang-bang decoupling 378
15.3 Combinatorial bounded strength decoupling 391
15.4 Conclusions and future directions 393

Part V Alternative quantum computation approaches 395

16 **Holonomic quantum computation** 397
Paolo Zanardi
16.1 Introduction 397
16.2 Holonomic quantum computation 398
16.3 HQC with quantum dots 400
16.4 Robustness 403
16.5 Hybridizing HQC and error-avoiding/correcting techniques 406
16.6 Conclusions 407
Appendix: quantum holonomies 408

17 **Fault tolerance for holonomic quantum computation** 412
Ognyan Oreshkov, Todd A. Brun, and Daniel A. Lidar
17.1 Holonomic quantum computation on subsystems 413
17.2 FTHQC on stabilizer codes without additional qubits 415
17.3 Conclusion and outlook 430

18 **Fault-tolerant measurement-based quantum computing** 432
Debbie Leung
18.1 Introduction 432
18.2 Common models for measurement-based quantum computation 433
18.3 Apparent issues concerning fault tolerance in measurement-based quantum computation 442
18.4 Simulation of an operation 442
18.5 Fault tolerance in graph state model 444
18.6 History and other approaches 451

Part VI Topological methods 453

19 **Topological codes** 455
Héctor Bombín
19.1 Introduction 455
19.2 Local codes 455
19.3 Surface homology 457
19.4 Surface codes 461
19.5 Color codes 471
19.6 Conclusions 480
19.7 History and further reading 480

20 **Fault-tolerant topological cluster state quantum computing** 482
Austin Fowler and Kovid Goyal
20.1 Introduction 482
20.2 Topological cluster states 482
20.3 Logical initialization and measurement 486
20.4 State injection 489
20.5 Logical gates 491
20.6 Topological cluster state error correction 492
20.7 Threshold 496
20.8 Overhead 499

Part VII Applications and implementations 507

21 **Experimental quantum error correction** 509
Dave Bacon
21.1 Experiments in liquid-state NMR 509
21.2 Ion trap quantum error correction 514
21.3 Experiments using linear optics quantum computation 517
21.4 The future of experimental quantum error correction 518

22 **Experimental dynamical decoupling** 519
Lorenza Viola
22.1 Introduction and overview 519
22.2 Single-axis decoupling 520
22.3 Two-axis decoupling 532
22.4 Recent experimental progress and outlook 534

23 **Architectures** 537
Jacob Taylor
23.1 The principles of fault tolerance 537
23.2 Memory 539
23.3 Building gates 543
23.4 Entangling operations and transport 545
23.5 Quantum networking 550

24 **Error correction in quantum communication** 553
Mark Wilde
24.1 Introduction 553
24.2 Entanglement distillation 554
24.3 Quantum key expansion 562
24.4 Continuous-variable quantum error correction 570
24.5 Implementations of quantum error correction for communication 578
24.6 Closing remarks 579
24.7 Historical notes 579

Part VIII Critical evaluation of fault tolerance 583

25 **Hamiltonian methods in quantum error correction and fault tolerance** 585
Eduardo Novais, Eduardo R. Mucciolo, and Harold U. Baranger
25.1 Introduction 585
25.2 Microscopic Hamiltonian models 588
25.3 Time evolution with quantum error correction 590
25.4 The threshold theorem in a critical environment 598
25.5 The threshold theorem and quantum phase transitions 599
25.6 An example: the simplified spin-boson model 601
25.7 Conclusions 609
Some useful results 609

26 **Critique of fault-tolerant quantum information processing** 612
Robert Alicki
26.1 Introduction 612
26.2 Fault-tolerant quantum computation 613
26.3 Fault tolerance and quantum memory 619
26.4 Concluding remarks 624

References 625
Index 657