

Xiaoying Zhuang

Meshless Methods for 3D Fracture Modelling with Level Sets

**A numerical framework for modelling arbitrary
crack propagation in 3D**

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