
Analytical Heat Transfer

Je-Chin Han



CRC Press

Taylor & Francis Group

Boca Raton London New York

CRC Press is an imprint of the
Taylor & Francis Group, an **Informa** business

Contents

| | |
|--|-----------|
| Preface | xi |
| 1. Heat Conduction Equations | 1 |
| 1.1 Introduction | 1 |
| 1.1.1 Conduction | 1 |
| 1.1.1.1 Fourier's Conduction Law | 1 |
| 1.1.2 Convection | 2 |
| 1.1.2.1 Newton's Cooling Law | 2 |
| 1.1.3 Radiation | 3 |
| 1.1.3.1 Stefan-Boltzmann Law | 4 |
| 1.1.4 Combined Modes of Heat Transfer | 4 |
| 1.2 General Heat Conduction Equations | 6 |
| 1.2.1 Derivations of General Heat Conduction Equations | 6 |
| 1.3 Boundary and Initial Conditions | 9 |
| 1.3.1 Boundary Conditions | 9 |
| 1.3.2 Initial Conditions | 9 |
| 1.4 Simplified Heat Conduction Equations | 10 |
| Problems | 12 |
| Reference | 12 |
| 2. 1-D Steady-State Heat Conduction | 13 |
| 2.1 Conduction through Plane Walls | 13 |
| 2.1.1 Conduction through Circular Tube Walls | 15 |
| 2.1.2 Critical Radius of Insulation | 17 |
| 2.2 Conduction with Heat Generation | 18 |
| 2.3 Conduction through Fins with Uniform Cross-Sectional Area | 21 |
| 2.3.1 Fin Performance | 25 |
| 2.3.1.1 Fin Effectiveness | 25 |
| 2.3.1.2 Fin Efficiency | 25 |
| 2.3.2 Radiation Effect | 26 |
| 2.4 Conduction through Fins with Variable Cross-Sectional Area: Bessel Function Solutions | 27 |
| 2.4.1 Radiation Effect | 30 |
| Problems | 38 |
| References | 43 |

| | |
|---|-----------|
| 3. 2-D Steady-State Heat Conduction | 45 |
| 3.1 Method of Separation of Variables: Given Temperature BC | 45 |
| 3.2 Method of Separation of Variables: Given Heat Flux and Convection BCs | 49 |
| 3.2.1 Given Surface Heat Flux BC | 49 |
| 3.2.2 Given Surface Convection BC | 51 |
| 3.3 Principle of Superposition for Nonhomogeneous BCs Superposition | 52 |
| 3.3.1 2-D Heat Conduction in Cylindrical Coordinates | 53 |
| 3.4 Principle of Superposition for Multidimensional Heat Conduction and for Nonhomogeneous Equations | 54 |
| 3.4.1 3-D Heat Conduction Problem | 54 |
| 3.4.2 Nonhomogeneous Heat Conduction Problem | 56 |
| Problems | 64 |
| References | 68 |
| 4. Transient Heat Conduction | 69 |
| 4.1 Method of Lumped Capacitance for 0-D Problems | 70 |
| 4.1.1 Radiation Effect | 72 |
| 4.2 Method of Separation of Variables for 1-D and for Multidimensional Transient Conduction Problems | 73 |
| 4.2.1 1-D Transient Heat Conduction in a Slab | 73 |
| 4.2.2 Multidimensional Transient Heat Conduction in a Slab (2-D or 3-D) | 75 |
| 4.2.3 1-D Transient Heat Conduction in a Rectangle with Heat Generation | 76 |
| 4.3 1-D Transient Heat Conduction in a Semiinfinite Solid Material | 78 |
| 4.3.1 Similarity Method for Semiinfinite Solid Material | 78 |
| 4.3.2 Laplace Transform Method for Semiinfinite Solid Material | 81 |
| 4.3.3 Approximate Integral Method for Semiinfinite Solid Material | 85 |
| 4.4 Heat Conduction with Moving Boundaries | 86 |
| 4.4.1 Freezing and Solidification Problems Using the Similarity Method | 87 |
| 4.4.2 Melting and Ablation Problems Using the Approximate Integral Method | 89 |
| 4.4.2.1 Ablation | 91 |
| Problems | 99 |
| References | 104 |

| | |
|--|-----|
| 5. Numerical Analysis in Heat Conduction | 105 |
| 5.1 Finite-Difference Energy Balance Method for 2-D Steady-State Heat Conduction | 105 |
| 5.2 Finite-Difference Energy Balance Method for 1-D Transient Heat Conduction | 114 |
| 5.2.1 Finite-Difference Explicit Method | 114 |
| 5.2.2 Finite-Difference Implicit Method | 117 |
| 5.3 2-D Transient Heat Conduction | 117 |
| Problems | 121 |
| References | 124 |
| 6. Heat Convection Equations | 125 |
| 6.1 Boundary-Layer Concepts | 125 |
| 6.2 General Heat Convection Equations | 130 |
| 6.3 2-D Heat Convection Equations | 131 |
| 6.4 Boundary-Layer Approximations | 135 |
| 6.4.1 Boundary-Layer Similarity/Dimensional Analysis | 136 |
| 6.4.2 Reynolds Analogy | 138 |
| Problems | 139 |
| References | 140 |
| 7. External Forced Convection | 141 |
| 7.1 Laminar Flow and Heat Transfer over a Flat Surface: Similarity Solution | 141 |
| 7.1.1 Summary of the Similarity Solution for Laminar Boundary-Layer Flow and Heat Transfer over a Flat Surface | 148 |
| 7.2 Laminar Flow and Heat Transfer over a Flat Surface: Integral Method | 150 |
| 7.2.1 Momentum Integral Equation by Von Karman | 150 |
| 7.2.2 Energy Integral Equation by Pohlhausen | 152 |
| 7.2.3 Outline of the Integral Approximate Method | 153 |
| Problems | 157 |
| References | 166 |
| 8. Internal Forced Convection | 167 |
| 8.1 Velocity and Temperature Profiles in a Circular Tube or between Parallel Plates | 167 |
| 8.2 Fully Developed Laminar Flow and Heat Transfer in a Circular Tube or between Parallel Plates | 169 |
| 8.2.1 Fully Developed Flow in a Tube: Friction Factor | 171 |
| 8.2.2 Case 1: Uniform Wall Heat Flux | 172 |
| 8.2.3 Case 2: Uniform Wall Temperature | 175 |
| Problems | 180 |
| References | 184 |

| | |
|--|-----|
| 9. Natural Convection | 185 |
| 9.1 Laminar Natural Convection on a Vertical Wall: Similarity Solution | 185 |
| 9.2 Laminar Natural Convection on a Vertical Wall: Integral Method | 190 |
| Problems | 193 |
| References | 194 |
| 10. Turbulent Flow Heat Transfer | 195 |
| 10.1 Reynolds-Averaged Navier–Stokes (RANS) Equation | 195 |
| 10.1.1 Continuity Equation | 197 |
| 10.1.2 Momentum Equation: RANS | 198 |
| 10.1.3 Enthalpy/Energy Equation | 199 |
| 10.1.4 Concept of Eddy or Turbulent Diffusivity | 200 |
| 10.1.5 Reynolds Analogy for Turbulent Flow | 203 |
| 10.2 Prandtl Mixing Length Theory and Law of Wall for Velocity and Temperature Profiles | 205 |
| 10.3 Turbulent Flow Heat Transfer | 208 |
| Problems | 216 |
| References | 219 |
| 11. Fundamental Radiation | 221 |
| 11.1 Thermal Radiation Intensity and Emissive Power | 221 |
| 11.2 Surface Radiation Properties for Blackbody and Real-Surface Radiation | 223 |
| 11.3 Solar and Atmospheric Radiation | 231 |
| Problems | 235 |
| References | 238 |
| 12. View Factor | 239 |
| 12.1 View Factor | 239 |
| 12.2 Evaluation of View Factor | 243 |
| 12.2.1 Method 1—Hottel’s Crossed-String Method for 2-D Geometry | 243 |
| 12.2.2 Method 2—Double-Area Integration | 250 |
| 12.2.3 Method 3—Contour Integration | 251 |
| 12.2.4 Method 4—Algebraic Method | 253 |
| Problems | 256 |
| References | 256 |
| 13. Radiation Exchange in a Nonparticipating Medium | 257 |
| 13.1 Radiation Exchange between Gray Diffuse Isothermal Surfaces in an Enclosure | 257 |
| 13.1.1 Method 1: Electric Network Analogy | 259 |
| 13.1.2 Method 2: Matrix Linear Equations | 263 |

| | |
|---|------------|
| 13.2 Radiation Exchange between Gray Diffuse Nonisothermal Surfaces | 268 |
| 13.3 Radiation Exchange between Nongray Diffuse Isothermal Surfaces | 270 |
| 13.4 Radiation Interchange among Diffuse and Nondiffuse (Specular) Surfaces | 270 |
| 13.5 Energy Balance in an Enclosure with Diffuse and Specular Surface | 271 |
| Problems | 272 |
| References | 274 |
| 14. Radiation Transfer through Gases | 275 |
| 14.1 Gas Radiation Properties | 275 |
| 14.1.1 Volumetric Absorption | 277 |
| 14.1.2 Geometry of Gas Radiation: Geometric Mean Beam Length | 279 |
| 14.2 Radiation Exchange between an Isothermal Gray Gas and Gray Diffuse Isothermal Surfaces in an Enclosure | 282 |
| 14.2.1 Matrix Linear Equations | 284 |
| 14.2.2 Electric Network Analogy | 285 |
| 14.3 Radiation Transfer through Gases with Nonuniform Temperature | 289 |
| 14.3.1 Cryogenic Thermal Insulation | 289 |
| 14.3.2 Radiation Transport Equation in the Participating Medium | 290 |
| Problems | 293 |
| References | 295 |
| Appendix A: Mathematical Relations and Functions | 297 |
| A.1 Useful Formulas | 297 |
| A.2 Hyperbolic Functions | 298 |
| A.3 Bessel Functions | 299 |
| A.3.1 Bessel Functions and Properties | 299 |
| A.3.2 Bessel Functions of the First Kind | 300 |
| A.3.3 Modified Bessel Functions of the First and Second Kinds | 301 |
| A.4 Gaussian Error Function | 302 |
| References | 303 |
| Index | 305 |