



The Mathematics of Open Quantum Systems

**Dissipative and Non-Unitary Representations
and Quantum Measurements**

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