



**TECHNISCHE  
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Institut für Geotechnik

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MITTEILUNGEN  
HEFT 26

**An extended bounding surface  
model for the application to  
general stress paths in sand**

**Ein erweitertes Bounding  
Surface Modell für die  
Anwendung auf allgemeine  
Spannungspfade im Sand**

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