Mathematics of Harmony as a New Interdisciplinary Direction and "Golden" Paradigm of Modern Science

Volume 1
The Golden Section, Fibonacci Numbers, Pascal Triangle, and Platonic Solids
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