



FRIEDRICH-SCHILLER-  
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# Optimal Stopping Problems with Expectation Constraints

## DISSERTATION

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