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# Fundamentals of Optimization

Methods, Minimum Principles,  
and Applications for Making Things Better

 Springer

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