

# Lecture Notes in Mathematics

A collection of informal reports and seminars

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The Multiplier Problem

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