

Inverse Problem Theory

and Methods for Model Parameter Estimation

Albert Tarantola

Institut de Physique du Globe de Paris
Université de Paris 6
Paris, France

siam

Society for Industrial and Applied Mathematics
Philadelphia

Contents

Preface	xi
1 The General Discrete Inverse Problem	1
1.1 Model Space and Data Space	1
1.2 States of Information	6
1.3 Forward Problem	20
1.4 Measurements and A Priori Information	24
1.5 Defining the Solution of the Inverse Problem	32
1.6 Using the Solution of the Inverse Problem	37
2 Monte Carlo Methods	41
2.1 Introduction	41
2.2 The Movie Strategy for Inverse Problems	44
2.3 Sampling Methods	48
2.4 Monte Carlo Solution to Inverse Problems	51
2.5 Simulated Annealing	54
3 The Least-Squares Criterion	57
3.1 Preamble: The Mathematics of Linear Spaces	57
3.2 The Least-Squares Problem	62
3.3 Estimating Posterior Uncertainties	70
3.4 Least-Squares Gradient and Hessian	75
4 Least-Absolute-Values Criterion and Minimax Criterion	81
4.1 Introduction	81
4.2 Preamble: ℓ_p -Norms	82
4.3 The ℓ_p -Norm Problem	86
4.4 The ℓ_1 -Norm Criterion for Inverse Problems	89
4.5 The ℓ_∞ -Norm Criterion for Inverse Problems	96
5 Functional Inverse Problems	101
5.1 Random Functions	101
5.2 Solution of General Inverse Problems	108
5.3 Introduction to Functional Least Squares	108
5.4 Derivative and Transpose Operators in Functional Spaces	119

5.5	General Least-Squares Inversion	133
5.6	Example: X-Ray Tomography as an Inverse Problem	140
5.7	Example: Travel-Time Tomography	143
5.8	Example: Nonlinear Inversion of Elastic Waveforms	144
6	Appendices	159
6.1	Volumetric Probability and Probability Density	159
6.2	Homogeneous Probability Distributions	160
6.3	Homogeneous Distribution for Elastic Parameters	164
6.4	Homogeneous Distribution for Second-Rank Tensors	170
6.5	Central Estimators and Estimators of Dispersion	170
6.6	Generalized Gaussian	174
6.7	Log-Normal Probability Density	175
6.8	Chi-Squared Probability Density	177
6.9	Monte Carlo Method of Numerical Integration	179
6.10	Sequential Random Realization	181
6.11	Cascaded Metropolis Algorithm	182
6.12	Distance and Norm	183
6.13	The Different Meanings of the Word Kernel	183
6.14	Transpose and Adjoint of a Differential Operator	184
6.15	The Bayesian Viewpoint of Backus (1970)	190
6.16	The Method of Backus and Gilbert	191
6.17	Disjunction and Conjunction of Probabilities	195
6.18	Partition of Data into Subsets	197
6.19	Marginalizing in Linear Least Squares	200
6.20	Relative Information of Two Gaussians	201
6.21	Convolution of Two Gaussians	202
6.22	Gradient-Based Optimization Algorithms	203
6.23	Elements of Linear Programming	223
6.24	Spaces and Operators	230
6.25	Usual Functional Spaces	242
6.26	Maximum Entropy Probability Density	245
6.27	Two Properties of ℓ_p -Norms	246
6.28	Discrete Derivative Operator	247
6.29	Lagrange Parameters	249
6.30	Matrix Identities	249
6.31	Inverse of a Partitioned Matrix	250
6.32	Norm of the Generalized Gaussian	250
7	Problems	253
7.1	Estimation of the Epicentral Coordinates of a Seismic Event	253
7.2	Measuring the Acceleration of Gravity	256
7.3	Elementary Approach to Tomography	259
7.4	Linear Regression with Rounding Errors	266
7.5	Usual Least-Squares Regression	269
7.6	Least-Squares Regression with Uncertainties in Both Axes	273

7.7	Linear Regression with an Outlier	275
7.8	Condition Number and A Posteriori Uncertainties	279
7.9	Conjunction of Two Probability Distributions	285
7.10	Adjoint of a Covariance Operator	288
7.11	Problem 7.1 Revisited	289
7.12	Problem 7.3 Revisited	289
7.13	An Example of Partial Derivatives	290
7.14	Shapes of the ℓ_p -Norm Misfit Functions	290
7.15	Using the Simplex Method	293
7.16	Problem 7.7 Revisited	295
7.17	Geodetic Adjustment with Outliers	296
7.18	Inversion of Acoustic Waveforms	297
7.19	Using the Backus and Gilbert Method	304
7.20	The Coefficients in the Backus and Gilbert Method	308
7.21	The Norm Associated with the 1D Exponential Covariance	308
7.22	The Norm Associated with the 1D Random Walk	311
7.23	The Norm Associated with the 3D Exponential Covariance	313
	References and References for General Reading	317
	Index	333