

Finite Element Methods for Engineers

Roger T Fenner

Department of Mechanical Engineering

Imperial College of Science, Technology and Medicine

London



Imperial College Press

Contents

<i>Preface</i>	vii
<i>Notation</i>	ix
<i>Some Program Variable Names</i>	xv
1 Introduction and Structural Analysis	1
1.1 Computer Programming	2
1.2 Structural Analysis	3
1.3 Case Study: Bending of a Tapered Beam	8
2 Continuum Mechanics Problems	15
2.1 Continuum Mechanics Equations	15
2.2 Some Physical Problems	20
2.3 Classification of Partial Differential Equations	31
2.4 Methods for Solving Harmonic and Biharmonic Equations	33
3 Finite Element Analysis of Harmonic Problems	37
3.1 Derivation of the Element Stiffness Matrix	37
3.2 Assembly of the Overall Stiffness Matrix	43
3.3 Comparison with the Finite Difference Approach	45
3.4 Variational Formulation	48
3.5 Boundary Conditions	53
3.6 Solution of the Linear Equations	55
3.7 Convergence of Finite Element Methods	61
3.8 A Computer Program for Harmonic Problems	63
4 Finite Element Meshes	71
4.1 Choice of Mesh	71
4.2 Mesh Data in Numerical Form	72

4.3	Generation of Mesh Data	74
4.4	Mesh Modification	86
5	Some Harmonic Problems	89
5.1	Case Study: Downstream Viscous Flow in a Rectangular Channel	89
5.2	Case Study: Torsion of Prismatic Bars	100
6	Finite Element Analysis of Biharmonic Problems	104
6.1	Derivation of the Element Stiffness Matrix	104
6.2	Assembly of the Overall Stiffness Matrix	110
6.3	Variational Formulation	112
6.4	Solution of the Linear Equations	113
6.5	Boundary Conditions	115
6.6	A Computer Program for Problems of the Biharmonic Plane Strain or Plane Stress Type	118
7	Some Biharmonic Problems	131
7.1	Case Study: Plane Strain Compression	131
7.2	Case Study: Stresses in Concentric Cylinders	135
7.3	Case Study: Stress Concentration near a Hole in a Flat Plate	138
8	Further Applications	148
8.1	Axi-symmetric Problems	148
8.2	Higher-order Elements	151
8.3	Three-dimensional Problems	153
8.4	Biharmonic Problems Involving Incompressible Materials	154
8.5	Plate and Shell Problems	155
8.6	Isoparametric Elements	156
8.7	Nonlinear Problems	156
8.8	A Summary of the Finite Element Approach	157
8.9	Concluding Remarks	158
<i>Appendix A</i>	Gaussian Elimination	159
<i>Appendix B</i>	The Gauss–Seidel Method	164
<i>Bibliography</i>		166
<i>Index</i>		167