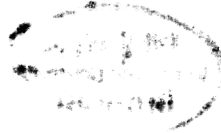


Michael Struwe

Variational Methods

Applications to Nonlinear
Partial Differential Equations
and Hamiltonian Systems



Second, Revised and
Substantially Expanded Edition

With 16 Figures



Springer

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