

# QUANTUM THEORY OF THE OPTICAL AND ELECTRONIC PROPERTIES OF SEMICONDUCTORS

*Second Edition*

**Hartmut Haug**

Institut für Theoretische Physik, Universität Frankfurt,  
Robert-Mayer-Str. 8, 6000 Frankfurt/Main,  
Germany

**Stephan W. Koch**

Optical Sciences Center & Department of Physics,  
University of Arizona, Tucson, AZ 85721, USA



**World Scientific**

Singapore • New Jersey • London • Hong Kong

# CONTENTS

1. Oscillator Model . . . . .	1
1-1. Optical Susceptibility . . . . .	2
1-2. Absorption and Refraction . . . . .	7
1-3. Retarded Green's Function . . . . .	12
2. Atom in a Classical Light Field . . . . .	17
2-1. Atomic Optical Susceptibility . . . . .	17
2-2. Oscillator Strength . . . . .	21
2-3. Optical Stark Shift . . . . .	24
3. Periodic Lattice of Atoms . . . . .	30
3-1. Reciprocal Lattice, Bloch Theorem . . . . .	30
3-2. Tight-Binding Approximation . . . . .	38
3-3. $\mathbf{k}\cdot\mathbf{p}$ Theory . . . . .	44
3-4. Degenerate Valence Bands . . . . .	48
4. Mesoscopic Semiconductor Structures . . . . .	56
4-1. Envelope Function Approximation . . . . .	57
4-2. Conduction Band Electrons in Quantum Wells . . . . .	59
4-3. Degenerate Hole Bands in Quantum Wells . . . . .	63
5. Free Carrier Transitions . . . . .	69
5-1. Optical Dipole Transitions . . . . .	69
5-2. Kinetics of Optical Interband Transitions . . . . .	74
5-3. Coherent Regime: Optical Bloch Equations . . . . .	77
5-4. Quasi-Equilibrium Regime: Free Carrier Absorption . . . . .	81

6. Ideal Quantum Gases . . . . .	90
6-1. The Ideal Fermi Gas . . . . .	91
6-2. The Ideal Bose Gas . . . . .	100
6-3. Ideal Quantum Gases in $d$ Dimensions. . . . .	104
7. Interacting Electron Gas . . . . .	111
7-1. The Electron Gas Hamiltonian. . . . .	111
7-2. Three-Dimensional Electron Gas . . . . .	118
7-3. Two-Dimensional Electron Gas . . . . .	124
7-4. Quasi-One-Dimensional Electron Gas . . . . .	128
8. Plasmons and Plasma Screening. . . . .	133
8-1. Plasmons and Pair Excitations . . . . .	133
8-2. Plasma Screening . . . . .	142
8-3. Analysis of the Lindhard Formula . . . . .	145
8-4. Plasmon-Pole Approximation . . . . .	152
9. Retarded Green's Function for Electrons . . . . .	155
9-1. Definitions . . . . .	155
9-2. Interacting Electron Gas . . . . .	157
9-3. Screened Hartree-Fock Approximation . . . . .	162
10. Excitons . . . . .	169
10-1. The Interband Polarization . . . . .	169
10-2. Wannier Equation . . . . .	176
10-3. Excitons . . . . .	180
10-4. The Ionization Continuum . . . . .	188
10-5. Optical Spectra. . . . .	191
11. Polaritons . . . . .	202
11-1. Dielectric Theory of Polaritons . . . . .	202
11-2. Hamiltonian Theory of Polaritons . . . . .	209

12. Semiconductor Bloch Equations . . . . .	218
12-1. Hamiltonian Equations . . . . .	218
12-2. Low Excitation Coherent Regime . . . . .	224
12-3. Collision Terms . . . . .	227
12-4. High Excitation Quasi-Equilibrium Regime . . . . .	232
13. Optical Quasi-Equilibrium Nonlinearities . . . . .	240
13-1. Numerical Matrix Inversion . . . . .	240
13-2. High-Density Approximations . . . . .	251
13-3. Effective Pair-Equation Approximation . . . . .	255
14. Optical Bistability . . . . .	264
14-1. The Light Field Equation . . . . .	265
14-2. The Carrier Equation . . . . .	268
14-3. Bistability in Semiconductor Resonators . . . . .	270
14-4. Intrinsic Optical Bistability . . . . .	276
15. The Semiconductor Laser . . . . .	283
15-1. Material Equations . . . . .	284
15-2. Field Equations . . . . .	286
15-3. Quantum Mechanical Langevin Equations . . . . .	290
15-4. Stochastic Laser Theory . . . . .	298
16. Coherent Effects in Semiconductors . . . . .	306
16-1. Excitonic Optical Stark Effect: Stationary Results . . . . .	307
16-2. Excitonic Optical Stark Effect: Dynamic Results . . . . .	317
16-3. Semiconductor Photon Echo . . . . .	327
17. Free-Carrier Electroabsorption . . . . .	336
17-1. Bulk Semiconductors . . . . .	336
17-2. Quantum Wells . . . . .	343

18. Exciton Electroabsorption . . . . .	350
18-1. Bulk Semiconductors . . . . .	350
18-2. Quantum Wells . . . . .	359
19. Magneto - Optics . . . . .	364
19-1. 2D Electron in a Perpendicular Magnetic Field . . . . .	364
19-2. 2D Magneto - Excitons . . . . .	369
19-3. 2D Magneto - Plasma . . . . .	375
20. Semiconductor Quantum Dots . . . . .	381
20-1. Effective Mass Approximation . . . . .	381
20-2. Single Particle Properties . . . . .	384
20-3. Pair States . . . . .	387
20-4. Dipole Transitions . . . . .	391
20-5. Optical Spectra . . . . .	395
21. Kinetics with Phonon Scattering . . . . .	400
21-1. Particle Propagator and Reduced Density Matrix . . . . .	400
21-2. Kinetic Equation . . . . .	404
21-3. Electron-LO Phonon Interaction . . . . .	405
21-4. Collision Integral and Intraband Kinetics . . . . .	407
21-5. Interband Kinetics with LO-Phonons . . . . .	415
Appendix A: Field Quantization . . . . .	422
A-1. Lagrange Functional . . . . .	422
A-2. Canonical Momentum and Hamilton Function . . . . .	428
A-3. Quantization of the Fields . . . . .	430

Appendix B: Nonequilibrium Green's Functions . . . . .	439
B-1. Contour Time Ordering and Matrix Propagators . . . . .	439
B-2. Free Electron Green's Function . . . . .	448
B-3. Dyson Equations . . . . .	451
B-4. Feynman Rules . . . . .	453
B-5. Equation of Motion for the Particle Propagator . . . . .	457
B-6. Electron Quantum Kinetics due to LO-Phonon Scattering . .	460
Index . . . . .	466