

# HANDBUCH DER PHYSIK

HERAUSGEGEBEN VON

S. FLÜGGE

BAND VIa/1

FESTKÖRPERMECHANIK I

BANDHERAUSGEBER

C. TRUESDELL

MIT 481 FIGUREN



SPRINGER-VERLAG  
BERLIN · HEIDELBERG · NEW YORK

1973

# Contents.

<b>The Experimental Foundations of Solid Mechanics.</b> By Professor JAMES F. BELL, The Johns Hopkins University, Baltimore, Maryland (USA). (With 481 Figures) . . .		1
I. Introduction . . . . .		1
II. Small deformation nonlinearity . . . . .		10
2.1. Introduction . . . . .		10
2.2. Nonlinear vs linear elasticity in the 18th century . . . . .		13
2.3. The small deformation nonlinearity of wood: Dupin (1815) . . . . .		15
2.4. Dupin's 18th century predecessors: Buffon (1741), Duhamel (1742), and Gauthey (1774) . . . . .		16
2.5. Details of Dupin's experiments on wooden beams (1815) . . . . .		18
2.6. Experiments on the nonlinear response of wood, iron, and stone, and the introduction of the concept of microplasticity: Hodgkinson (1824-1844)		22
2.7. "Gerstner's law" for steel piano wire (1824) . . . . .		30
2.8. The discovery of the creep of metals: Coriolis and Vicat (1830-1834) . . . .		32
2.9. The first resolution of microstrain: Vicat (1831) . . . . .		34
2.10. Experiments on the stability of the permanent deformation of iron wire: Leblanc (1839) . . . . .		38
2.11. On the phenomenon discovered by Savart (1837) and Masson (1841), now known as the Portevin-Le Chatelier (1923) effect . . . . .		41
2.12. The experiments of Gough (1805) and Wilhelm Weber (1830) introducing thermoelasticity; Weber's discovery of the elastic after-effect (1835) . . . .		44
2.13. The large deformation of gut strings: Karmarsch (1841) . . . . .		52
2.14. Experiments on the elasticity and cohesion of the principal tissues of the human body: Wertheim (1846-1847) . . . . .		56
2.15. Further experiments on the elasticity of organic tissue: The comparison of response functions for live and dead specimens. Wundt (1858), Volkmann (1859) . . . . .		62
2.16. The repeal of Hooke's law by the British Royal Iron Commission in 1849 .		70
2.17. Experiments on stress relaxation in glass and brass: The origins of non- linear viscoelasticity. Kohlrausch (1863). . . . .		74
2.18. On the change of volume during plastic deformation: The experiments of Bauschinger (1879) . . . . .		83
2.19. Nonlinear torsion including influence on magnetization, 1857 to 1881 . .		87
2.20. The decrease of moduli with permanent deformation: The experiments on metals of Wertheim (1844-1848), Kelvin (1865), Tomlinson (1881), and Fischer (1882) . . . . .		92
2.21. The cyclical loading of raw silk: Müller (1882) . . . . .		97
2.22. The first precise measurement of the nonlinearity of metals for infinitesimal deformation: Joseph Thompson (1891) . . . . .		101
2.23. Hartig's nonlinear law: A general response function for the small deforma- tion of solids (1893) . . . . .		105
2.24. The Bach-Schüle law (1897): A rediscovery of the parabolic response function of James Bernoulli (1695) and of Hodgkinson (1824) . . . . .		111
2.25. Grüneisen's experiments (1906) using an interferometer, which established Hartig's law for the infinitesimal deformation of metals . . . . .		116
2.26. On some examples of unwitting rediscovery in the 20th century of non- linear phenomena first observed in the 19th century . . . . .		125
2.26a. A law for paints and varnish: Nelson (1921) . . . . .		126
2.26b. Sayre's nonlinear law for the small deformation of steel (1930) . . . . .		127
2.26c. Nonlinearity in tensile experiments on copper alloys: Smith (1940-1948) .		130
2.26d. An exhaustive study of a single solid in simple loading: The analysis of the small deformation of beryllium copper by Richards (1952). . . . .		132

2.26e. Hodgkinson's parabola and "elastic defect": The microplasticity experiments of Thomas and Averbach (1959) and of Bilello and Metzger (1969)	140
2.26f. A comparison of the response of fibre and whole muscle: The experiments of Sichel (1935)	142
2.26g. The nonlinear response of artificial stone: The experiments of Powers (1938)	143
2.26h. The "after-effect" in lead single crystals: Chalmers (1935)	145
2.26i. The decrease in $E$ with micro-permanent deformation: Lauriente and Pond's experiments on aluminum crystals (1956)	147
2.27. Some recent experiments on the nonlinearity of infinitesimal deformation in crystalline solids	148
2.28. New problems for the critic in reviewing experiments described in the literature during the past decade	153
2.29. Summary	155
III. Small deformation: The linear approximation	156
3.1. The 17th century origins: Hooke and Mariotte	156
3.2. Experiments before 1780: Riccati, Musschenbroek, s' Gravesande, Coulomb; Euler's introduction of the concept of an elastic modulus	160
3.3. The origins of an experimental science of solid mechanics: The torsion studies of Coulomb in the 1780's	168
3.4. Coulomb's first measurement of an elastic modulus and his experiments on viscosity and plasticity (1784)	173
3.5. On the measurement of elastic constants	179
3.6. The experiments of Chladni on the longitudinal vibration of bars (1787)	182
3.7. An assessment of fact and myth for the modulus in Young's Lectures on Natural Philosophy (1807)	184
3.8. Biot's use of the new Paris water pipes to obtain the first direct measurement of the velocity of sound in a solid (1809)	191
3.9. Duleau's introduction of quasi-static measurements into the study of linear elasticity (1813)	196
3.10. Research on elastic moduli in the three decades (1811-1841) before Wertheim	205
3.11. Guillaume Wertheim: A Faraday without a Maxwell	218
3.12. Wertheim's memoir of 1842: Values of $E$ for 15 elements and the first study of the effect of ambient temperature, prior history of the specimen, rate of loading, and atomic spacing	220
3.13. Wertheim's memoir of 1843: The first experiments on binary and tertiary alloys including, for 64 combinations, the influence upon $E$ of composition and rate of loading	230
3.14. Wertheim's memoir of 1844: The first study of the dependence of $E$ upon the strength of electric and magnetic fields	238
3.15. Wertheim's memoirs in 1845-1846 on the elasticity of glass, wood, and human tissue	240
3.16. Wertheim's first experiments on Poisson's ratio which revealed that the Poisson-Cauchy molecular theory failed to describe crystalline solids (1848)	245
3.17. Wertheim succeeds in making the first measurement of the frequency of standing waves in a liquid column (1848)	251
3.18. Wertheim on vibration of plates, and the "deep tone" of vibrating rods	254
3.19. The Wertheim controversy viewed from the 20th century	257
3.20. Kirchhoff's experiment for the direct measurement of Poisson's ratio (1859)	259
3.21. Cornu's optical interference experiment for determining Poisson's ratio (1869)	264
3.22. The experiments of Voigt on the isotropy and moduli of glass (1882)	269
3.23. Mercadier's determination of the ratio of elastic constants from the first and second mode frequencies of a vibrating plate (1888)	272
3.24. The piezometer experiments of Amagat (1884-1889)	274
3.25. The experiments of Bock on the dependence of Poisson's ratio upon temperature (1894)	278
3.26. Straubel's definitive study of the Cornu experiment for the direct measurement of Poisson's ratio (1899)	282
3.27. Grüneisen's experiments checking isotropic formulae by the independent measurement of $E$ , $\mu$ , and $\nu$	287

3.28. The mid-20th century repetition of Kirchhoff's experiment for determining Poisson's ratio . . . . .	293
3.29. The confusion generated by the experiments of Kupffer (1848-1863) . . . . .	296
3.30. The Mallock method for the quasi-static determination of the bulk modulus . . . . .	303
3.31. Grüneisen's use of Mallock's method to compare elastic constants in isotropic solids (1910) . . . . .	304
3.32. The linear approximation and one-dimensional wave propagation: Wertheim and Breguet (1851) . . . . .	306
3.33. Exner's experiments on wave propagation in rubber (1874) . . . . .	308
3.34. The axial collision of rods with an assumed linear response function: The Boltzmann experiment (1881 et seq.) vs Saint-Venant's theory (1867) . . . . .	313
3.35. Hausmaninger's use (1884) of the time of contact technique of Pouillet (1844) in the Boltzmann experiment, and the half century of similar experiments (1884-1936) . . . . .	315
3.36. The first use of electric resistance elements to study wave profiles in the Boltzmann experiment: Fanning and Bassett (1940) . . . . .	329
3.37. Davies' (1948) use of a capacitance displacement technique for the first comparison of pulse profiles with Pochhammer's (1876) three dimensional theory for cylindrical rods . . . . .	331
3.38. Experiments on the propagation of waves of small amplitude in metal cylinders during the past two decades: A sequence of changes in techniques and interpretation . . . . .	338
3.39. Ultrasonic determination of elastic constants . . . . .	352
3.40. Short-time loading histories . . . . .	357
3.41. On temperature dependence of elastic constants (1843-1910) . . . . .	360
3.42. A comparison of ultrasonic and quasi-static temperature coefficients . . . . .	377
3.43. On temperature dependence of elastic constants and damping coefficients, after 1910 . . . . .	380
3.44. The quantized distribution of elastic shear moduli at the zero point for isotropic bodies, and the multiple elasticities for a given isotropic solid: Bell (1964-1968) . . . . .	397
3.45. Anisotropy . . . . .	406
3.46. Thermoelasticity . . . . .	411
3.47. Viscoelasticity . . . . .	413
3.48. Summary . . . . .	417
IV. Finite Deformation . . . . .	419
4.1. Paucity of experiment before 1800 . . . . .	419
4.2. 1800 to 1850: The experiments on creep of Navier and Coriolis, and the summary by Poncelet of research before 1840 . . . . .	421
4.3. Tresca on the flow of solids (1864-1872) . . . . .	427
4.4. The punching and extrusion experiments of Tresca . . . . .	429
4.5. Thurston's discovery of the dependence of the elastic limit upon the previous stress history and the elapsed time (1873) . . . . .	449
4.6. Experiments on yield limits, elastic limits, and fatigue, preceding those of Thurston and Bauschinger: Thalén (1864), Wiedemann (1859), and Wöhler (1858-1870) . . . . .	457
4.7. The experiments of Bauschinger on the yield limit and the elastic limit (1875-1886) . . . . .	462
4.8. On the cohesion of solids under pressure: The experiments of Spring (1880) . . . . .	474
4.9. Early 20th century experiments on the flow of solids under high pressure: Tammann (1902) . . . . .	478
4.10. The beginning of the experimental study of the large deformation of crystalline solids responding to loading histories with more than one non-zero stress component: Guest (1900) . . . . .	483
4.11. The ductility of marble and sandstone when responding to a general state of stress: von Kármán (1911) . . . . .	486
4.12. The large deformation of solids under high hydrostatic stress: Bridgman (1909-1961) . . . . .	490
4.13. Dynamic response of solids under high pressure . . . . .	499
4.14. Further study of the Guest experiment: Lode (1926), and Taylor and Quinney (1931) . . . . .	501
4.15. On the relation between response functions for large deformation, for different radial loading paths: E. A. Davis' experiments on polycrystals (1943-1945) . . . . .	509

