

GOODNESS-OF-FIT TECHNIQUES

edited by

Ralph B. D'Agostino

Department of Mathematics
Boston University
Boston, Massachusetts

Michael A. Stephens

Department of Mathematics and Statistics
Simon Fraser University
Burnaby, British Columbia, Canada

MARCEL DEKKER, INC.

New York and Basel

Contents

Preface	v
Acknowledgments	ix
Contributors	xviii
1. OVERVIEW	1
Ralph B. D'Agostino and Michael A. Stephens	
1.1 Goodness-of-Fit Techniques	1
1.2 Objectives of the Book	3
1.3 The Topics of the Book	4
2. GRAPHICAL ANALYSIS	7
Ralph B. D'Agostino	
2.1 Introduction	7
2.2 Empirical Cumulative Distribution Function	8
2.3 General Concepts of Probability Plotting	24
2.4 Normal Probability Plotting	35
2.5 Lognormal Probability Plotting	47
2.6 Weibull Probability Plotting	54
2.7 Other Topics	57
2.8 Concluding Comment	59
References	59
	xi

3. TESTS OF CHI-SQUARED TYPE	63
David S. Moore	
3.1 Introduction	63
3.2 Classical Chi-Squared Statistics	64
3.3 General Chi-Squared Statistics	75
3.4 Recommendations on Use of Chi-Squared Tests	91
References	93
4. TESTS BASED ON EDF STATISTICS	97
Michael A. Stephens	
4.1 Introduction	97
4.2 Empirical Distribution Function Statistics	97
4.3 Goodness-of-Fit Tests Based on the EDF (EDF Tests)	102
4.4 EDF Tests for a Fully Specified Distribution (Case 0)	104
4.5 Comments on EDF Tests for Case 0	106
4.6 Power of EDF Statistics for Case 0	110
4.7 EDF Tests for Censored Data: Case 0	111
4.8 EDF Tests for the Normal Distribution with Unknown Parameters	122
4.9 EDF Tests for the Exponential Distribution	133
4.10 EDF Tests for the Extreme-Value Distribution	145
4.11 EDF Tests for the Weibull Distribution	149
4.12 EDF Tests for the Gamma Distribution	151
4.13 EDF Tests for the Logistic Distribution	156
4.14 EDF Tests for the Cauchy Distribution	160
4.15 EDF Tests for the von Mises Distribution	164
4.16 EDF Tests for Continuous Distributions: Miscellaneous Topics	166
4.17 EDF Tests for Discrete Distributions	171
4.18 Combinations of Tests	176
4.19 EDF Statistics as Indicators of Parent Populations	180
4.20 Tests Based on Normalized Spacings	180
References	185
5. TESTS BASED ON REGRESSION AND CORRELATION	195
Michael A. Stephens	
5.1 Introduction	195
5.2 Regression Tests: Models	196
5.3 Measure of Fit	197
5.4 Tests Based on the Correlation Coefficient	198
5.5 The Correlation Tests for the Uniform Distribution with Unknown Limits	199

5.6	The Correlation Test for $U(0, 1)$	201
5.7	Regression Tests for the Normal Distribution 1	201
5.8	Regression Tests Based on Residuals	205
5.9	Tests Based on the Ratio of Two Estimates of Scale	206
5.10	Regression Tests for the Normal Distribution 2	207
5.11	Regression Tests for the Exponential Distribution	215
5.12	Tests Based on the Ratio of Two Estimates of Scale: Further Comments	223
5.13	Regression Tests for Other Distributions: General Comments	224
5.14	Correlation Tests for the Extreme-Value Distribution	225
5.15	Correlation Tests for Other Distributions References	225 230
6.	SOME TRANSFORMATION METHODS IN GOODNESS-OF-FIT Charles P. Quesenberry	235
6.1	Introduction	235
6.2	Probability Integral Transformations	239
6.3	Some Properties of CPIT's	244
6.4	Testing Simple Uniformity	246
6.5	Transformations for Particular Families	252
6.6	Numerical Examples References	260 275
7.	MOMENT ($\sqrt{b_1}$, b_2) TECHNIQUES K. O. Bowman and L. R. Shenton	279
7.1	Introduction	279
7.2	Normal Distribution	280
7.3	Nonnormal Sampling	287
7.4	Moments of Sample Moments	288
7.5	The Correlation Between $\sqrt{b_1}$ and b_2	292
7.6	Simultaneous Behavior of $\sqrt{b_1}$ and b_2	295
7.7	A Bivariate Model	306
7.8	Experimental Samples References	316 318
8.	TESTS FOR THE UNIFORM DISTRIBUTION Michael A. Stephens	331
8.1	Introduction	331
8.2	Notation	332
8.3	Transformations to Uniforms	332

8.4	Transformation from Uniforms to Uniforms	333
8.5	Superuniform Observations	334
8.6	Tests Based on the Empirical Distribution Function (EDF)	334
8.7	Regression and Correlation Tests	336
8.8	Other Tests Based on Order Statistics	336
8.9	Statistics Based on Spacings	338
8.10	Statistics for Special Alternatives	345
8.11	The Neyman-Barton Smooth Tests	351
8.12	Components of Test Statistics	355
8.13	The Effect on Test Statistics of Certain Patterns of U-Values	356
8.14	Power of Test Statistics	357
8.15	Statistics for Combining Independent Tests for Several Samples	357
8.16	Tests for a Uniform Distribution with Unknown Limits	360
8.17	Tests for Censored Uniform Samples	361
	References	361
9.	TESTS FOR THE NORMAL DISTRIBUTION	367
	Ralph B. D'Agostino	
9.1	Introduction	367
9.2	Complete Random Samples	368
9.3	Classification of Existing Tests	370
9.4	Comparisons of Tests	403
9.5	Recommendations	405
9.6	Tests of Normality on Residuals	406
9.7	Multivariate Normality	409
	References	413
10.	TESTS FOR THE EXPONENTIAL DISTRIBUTION	421
	Michael A. Stephens	
10.1	Introduction and Contents	421
10.2	Notation	424
10.3	Tests for Exponentiality: The Four Cases	425
10.4	Applications of the Exponential Distribution	426
10.5	Transformations from Exponentials to Exponentials or to Uniforms	429
10.6	Test Situations and Choice of Procedures	432
10.7	Tests with Origin Known: Groups 1, 2, and 3	435
10.8	Group 1 Tests	435
10.9	Group 2 Tests, Applied to $U = JX$	438

10.10	The Effect of Zero Values, and of Ties	444
10.11	Group 3 Tests Applied to $X' = NX$, or to $U' = KX$	445
10.12	Discussion of the Data Set	451
10.13	Evaluation of Tests for Exponentiality	451
10.14	Tests with Origin and Scale Unknown	455
10.15	Summary	456
	References	457
11.	ANALYSIS OF DATA FROM CENSORED SAMPLES	461
	John R. Michael and William R. Schucany	
11.1	Introduction	461
11.2	Probability Plots	463
11.3	Testing a Simple Null Hypothesis	480
11.4	Testing a Composite Hypothesis	487
	References	493
12.	THE ANALYSIS AND DETECTION OF OUTLIERS	497
	Gary L. Tietjen	
12.1	Introduction	497
12.2	A Single Outlier in a Univariate Sample	500
12.3	Multiple Outliers in a Univariate Sample	504
12.4	The Identification of a Single Outlier in Linear Models	507
12.5	Multiple Outliers in the Linear Model	516
12.6	Accommodation of Outliers	517
12.7	Multivariate Outliers	520
12.8	Outliers in Time Series	520
	References	521
	APPENDIX	523
1.	Table 1, Cumulative Distribution Function of the Standard Normal Distribution	524
2.	Table 2, Critical Values of the Chi-Square Distribution	526
3.	Simulated Data Sets	527
4.	Real Data Sets	546
	INDEX	551