

Vibrations and Waves in Continuous Mechanical Systems

Peter Hagedorn
TU Darmstadt, Germany

Anirvan DasGupta
IIT Kharagpur, India



John Wiley & Sons, Ltd

Contents

Preface	xi
1 Vibrations of strings and bars	1
1.1 Dynamics of strings and bars: the Newtonian formulation	1
1.1.1 Transverse dynamics of strings	1
1.1.2 Longitudinal dynamics of bars	6
1.1.3 Torsional dynamics of bars	7
1.2 Dynamics of strings and bars: the variational formulation	9
1.2.1 Transverse dynamics of strings	10
1.2.2 Longitudinal dynamics of bars	11
1.2.3 Torsional dynamics of bars	13
1.3 Free vibration problem: Bernoulli's solution	14
1.4 Modal analysis	18
1.4.1 The eigenvalue problem	18
1.4.2 Orthogonality of eigenfunctions	24
1.4.3 The expansion theorem	25
1.4.4 Systems with discrete elements	27
1.5 The initial value problem: solution using Laplace transform	30
1.6 Forced vibration analysis	31
1.6.1 Harmonic forcing	32
1.6.2 General forcing	36
1.7 Approximate methods for continuous systems	40
1.7.1 Rayleigh method	41
1.7.2 Rayleigh–Ritz method	43
1.7.3 Ritz method	44
1.7.4 Galerkin method	47
1.8 Continuous systems with damping	50
1.8.1 Systems with distributed damping	50
1.8.2 Systems with discrete damping	53
1.9 Non-homogeneous boundary conditions	56
1.10 Dynamics of axially translating strings	57
1.10.1 Equation of motion	58
1.10.2 Modal analysis and discretization	58

1.10.3	Interaction with discrete elements	61
	Exercises	62
	References	67
2	One-dimensional wave equation: d'Alembert's solution	69
2.1	D'Alembert's solution of the wave equation	69
2.1.1	The initial value problem	72
2.1.2	The initial value problem: solution using Fourier transform	76
2.2	Harmonic waves and wave impedance	77
2.3	Energetics of wave motion	79
2.4	Scattering of waves	83
2.4.1	Reflection at a boundary	83
2.4.2	Scattering at a finite impedance	87
2.5	Applications of the wave solution	93
2.5.1	Impulsive start of a bar	93
2.5.2	Step-forcing of a bar with boundary damping	95
2.5.3	Axial collision of bars	99
2.5.4	String on a compliant foundation	102
2.5.5	Axially translating string	104
	Exercises	107
	References	112
3	Vibrations of beams	113
3.1	Equation of motion	113
3.1.1	The Newtonian formulation	113
3.1.2	The variational formulation	116
3.1.3	Various boundary conditions for a beam	118
3.1.4	Taut string and tensioned beam	120
3.2	Free vibration problem	121
3.2.1	Modal analysis	121
3.2.2	The initial value problem	132
3.3	Forced vibration analysis	133
3.3.1	Eigenfunction expansion method	134
3.3.2	Approximate methods	135
3.4	Non-homogeneous boundary conditions	137
3.5	Dispersion relation and flexural waves in a uniform beam	138
3.5.1	Energy transport	140
3.5.2	Scattering of flexural waves	142
3.6	The Timoshenko beam	144
3.6.1	Equations of motion	144
3.6.2	Harmonic waves and dispersion relation	147
3.7	Damped vibration of beams	149
3.8	Special problems in vibrations of beams	151
3.8.1	Influence of axial force on dynamic stability	151
3.8.2	Beam with eccentric mass distribution	155
3.8.3	Problems involving the motion of material points of a vibrating beam	159

3.8.4	Dynamics of rotating shafts	163
3.8.5	Dynamics of axially translating beams	165
3.8.6	Dynamics of fluid-conveying pipes	168
	Exercises	171
	References	178
4	Vibrations of membranes	179
4.1	Dynamics of a membrane	179
4.1.1	Newtonian formulation	179
4.1.2	Variational formulation	182
4.2	Modal analysis	185
4.2.1	The rectangular membrane	185
4.2.2	The circular membrane	190
4.3	Forced vibration analysis	197
4.4	Applications: kettledrum and condenser microphone	197
4.4.1	Modal analysis	197
4.4.2	Forced vibration analysis	201
4.5	Waves in membranes	202
4.5.1	Waves in Cartesian coordinates	202
4.5.2	Waves in polar coordinates	204
4.5.3	Energetics of membrane waves	207
4.5.4	Initial value problem for infinite membranes	208
4.5.5	Reflection of plane waves	209
	Exercises	213
	References	214
5	Vibrations of plates	217
5.1	Dynamics of plates	217
5.1.1	Newtonian formulation	217
5.2	Vibrations of rectangular plates	222
5.2.1	Free vibrations	222
5.2.2	Orthogonality of plate eigenfunctions	228
5.2.3	Forced vibrations	229
5.3	Vibrations of circular plates	231
5.3.1	Free vibrations	231
5.3.2	Forced vibrations	234
5.4	Waves in plates	236
5.5	Plates with varying thickness	238
	Exercises	239
	References	241
6	Boundary value and eigenvalue problems in vibrations	243
6.1	Self-adjoint operators and eigenvalue problems for undamped free vibrations	243
6.1.1	General properties and expansion theorem	243
6.1.2	Green's functions and integral formulation of eigenvalue problems	252
6.1.3	Bounds for eigenvalues: Rayleigh's quotient and other methods	255
6.2	Forced vibrations	259

6.2.1	Equations of motion	259
6.2.2	Green's function for inhomogeneous vibration problems	260
6.3	Some discretization methods for free and forced vibrations	261
6.3.1	Expansion in function series	261
6.3.2	The collocation method	262
6.3.3	The method of subdomains	266
6.3.4	Galerkin's method	267
6.3.5	The Rayleigh–Ritz method	269
6.3.6	The finite-element method	272
	References	288
7	Waves in fluids	289
7.1	Acoustic waves in fluids	289
7.1.1	The acoustic wave equation	289
7.1.2	Planar acoustic waves	294
7.1.3	Energetics of planar acoustic waves	295
7.1.4	Reflection and refraction of planar acoustic waves	297
7.1.5	Spherical waves	300
7.1.6	Cylindrical waves	305
7.1.7	Acoustic radiation from membranes and plates	307
7.1.8	Waves in wave guides	314
7.1.9	Acoustic waves in a slightly viscous fluid	318
7.2	Surface waves in incompressible liquids	320
7.2.1	Dynamics of surface waves	320
7.2.2	Sloshing of liquids in tanks	323
7.2.3	Surface waves in a channel	330
	Exercises	334
	References	337
8	Waves in elastic continua	339
8.1	Equations of motion	339
8.2	Plane elastic waves in unbounded continua	344
8.3	Energetics of elastic waves	346
8.4	Reflection of elastic waves	348
8.4.1	Reflection from a free boundary	349
8.5	Rayleigh surface waves	353
8.6	Reflection and refraction of planar acoustic waves	357
	Exercises	359
	References	361
A	The variational formulation of dynamics	363
	References	365
B	Harmonic waves and dispersion relation	367
B.1	Fourier representation and harmonic waves	367
B.2	Phase velocity and group velocity	369
	References	372

C Variational formulation for dynamics of plates

373

References

378

Index

379