CONTENTS

PREFACE xvii

ACKNOWLEDGMENTS xxii

1 PRELIMINARY BACKGROUND 1

1.1 Maxwell's Equations 1
  1.1.1 Differential Representations 1
  1.1.2 Integral Representations 3
  1.1.3 Time Harmonic Forms 4
  1.1.4 Constitutive Relations 5
  1.1.5 Poynting Theorem and Lossless Conditions 6
  1.1.6 Duality Principle 9

1.2 Scalar Wave Equations 9
  1.2.1 Acoustic Wave Equation 10
  1.2.2 Scalar Wave Equation from Electromagnetics 12
  1.2.3 Cartesian Coordinates 12
  1.2.4 Cylindrical Coordinates 14
  1.2.5 Spherical Coordinates 16

1.3 Vector Wave Equations 17
  1.3.1 Boundary Conditions 18
  1.3.2 Reciprocity Theorem 20
  1.3.3 Plane Wave in Homogeneous, Anisotropic Media 22
  1.3.4 Green’s Function 24

1.4 Huygens' Principle 29
  1.4.1 Scalar Waves 29
  1.4.2 Electromagnetic Waves 31

1.5 Uniqueness Theorem 32
  1.5.1 Scalar Wave Equation 33
  1.5.2 Vector Wave Equation 35

Exercises for Chapter 1 37
References for Chapter 1 41
Further Readings for Chapter 1 42
## 2 PLANARLY LAYERED MEDIA

### 2.1 One-Dimensional Planar Inhomogeneity
- 2.1.1 Derivation of the Scalar Wave Equations
- 2.1.2 Reflection from a Half-Space
- 2.1.3 Reflection and Transmission in a Multilayered Medium
- 2.1.4 Ricatti Equation for Reflection Coefficients
- 2.1.5 Specific Inhomogeneous Profiles

### 2.2 Spectral Representations of Sources
- 2.2.1 A Line Source
- 2.2.2 A Point Source
- 2.2.3 Riemann Sheets and Branch Cuts

### 2.3 A Source on Top of a Layered Medium
- 2.3.1 Electric Dipole Fields
- 2.3.2 Magnetic Dipole Fields
- 2.3.3 The Transverse Field Components

### 2.4 A Source Embedded in a Layered Medium

### 2.5 Asymptotic Expansions of Integrals
- 2.5.1 Method of Stationary Phase
- 2.5.2 Method of Steepest Descent
- 2.5.3 Uniform Asymptotic Expansions

### 2.6 Dipole Over Layered Media—Asymptotic Expansions
- 2.6.1 Dipole Over Half-Space (VMD)
- 2.6.2 Dipole Over Half-Space (VED)
- 2.6.3 Dipole Over a Slab
- 2.6.4 Example of Uniform Asymptotic Expansion—Transmitted Wave in a Half-Space
- 2.6.5 Angular Spectrum Representation

### 2.7 Singularities of the Sommerfeld Integrals
- 2.7.1 Absence of Branch Points
- 2.7.2 Bounds on the Locations of Singularities
- 2.7.3 Numerical Integration of Sommerfeld Integrals

### 2.8 WKB Method
- 2.8.1 Derivation of the WKB Solution
- 2.8.2 Asymptotic Matching
CONTENTS

2.9 Propagator Matrix 128
  2.9.1 Derivation of the State Equation 129
  2.9.2 Solution of the State Equation 129
  2.9.3 Reflection from a Three-Layer Medium 130
  2.9.4 Reflection from an Inhomogeneous Slab 131

2.10 Waves in Anisotropic, Layered Media 133
  2.10.1 Derivation of the State Equation 133
  2.10.2 Solution of the State Equation 135
  2.10.3 Reflection from an Interface of Anisotropic Half Spaces 136
  2.10.4 Reflection from a Slab 137
  2.10.5 Geometrical Optics Series 138

Exercises for Chapter 2 140
References for Chapter 2 151
Further Readings for Chapter 2 155

3 CYLINDRICALLY AND SPHERICALLY LAYERED MEDIA 161

3.1 Cylindrically Layered Media—Single Interface Case 161
  3.1.1 Vector Wave Equation in Cylindrical Coordinates 162
  3.1.2 Reflection and Transmission of an Outgoing Wave 163
  3.1.3 Reflection and Transmission of a Standing Wave 165

3.2 Cylindrically Layered Media—Multi-Interface Case 167
  3.2.1 The Outgoing-Wave Case 167
  3.2.2 The Standing-Wave Case 170

3.3 Source in a Cylindrically Layered Medium 172
  3.3.1 Discrete, Angular-Wave-Number Representation 173
  3.3.2 Continuum, Angular-Wave-Number Representation 177

3.4 Propagator Matrix—Cylindrical Layers 179
  3.4.1 Isotropic, Layered Media 179
  3.4.2 Anisotropic, Layered Media 182

3.5 Spherically Layered Media—Single Interface Case 184
  3.5.1 Vector Wave Equation in Spherical Coordinates 185
  3.5.2 Reflection and Transmission of an Outgoing Wave 187
  3.5.3 Reflection and Transmission of a Standing Wave 189

3.6 Spherically Layered Media—Multi-Interface Case 191
<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.6.1</td>
<td>The Outgoing-Wave Case</td>
<td>191</td>
</tr>
<tr>
<td>3.6.2</td>
<td>The Standing-Wave Case</td>
<td>192</td>
</tr>
<tr>
<td>3.7</td>
<td>Source in a Spherically Layered Medium</td>
<td>193</td>
</tr>
<tr>
<td>3.8</td>
<td>Propagator Matrix—Spherical Layers</td>
<td>197</td>
</tr>
<tr>
<td></td>
<td>Exercises for Chapter 3</td>
<td>199</td>
</tr>
<tr>
<td></td>
<td>References for Chapter 3</td>
<td>204</td>
</tr>
<tr>
<td></td>
<td>Further Readings for Chapter 3</td>
<td>206</td>
</tr>
<tr>
<td>4</td>
<td>TRANSIENTS</td>
<td>211</td>
</tr>
<tr>
<td>4.1</td>
<td>Causality of Transient Response</td>
<td>211</td>
</tr>
<tr>
<td>4.1.1</td>
<td>The Kramers-Kronig Relation</td>
<td>212</td>
</tr>
<tr>
<td>4.1.2</td>
<td>Causality and Contour of Integration</td>
<td>214</td>
</tr>
<tr>
<td>4.2</td>
<td>The Cagniard-de Hoop Method</td>
<td>215</td>
</tr>
<tr>
<td>4.2.1</td>
<td>Line Source in Free-Space—Two-Dimensional Green’s Function</td>
<td>216</td>
</tr>
<tr>
<td>4.2.2</td>
<td>Point Source in Free-Space—Three-Dimensional Green’s Function</td>
<td>219</td>
</tr>
<tr>
<td>4.2.3</td>
<td>Line Source Over Half-Space—Transient Response</td>
<td>221</td>
</tr>
<tr>
<td>4.2.4</td>
<td>Dipole Over Half Space—Transient Response</td>
<td>224</td>
</tr>
<tr>
<td>4.3</td>
<td>Multi-interface Problems</td>
<td>227</td>
</tr>
<tr>
<td>4.4</td>
<td>Direct Inversion</td>
<td>228</td>
</tr>
<tr>
<td>4.5</td>
<td>Numerical Integration of Fourier Integrals</td>
<td>231</td>
</tr>
<tr>
<td>4.5.1</td>
<td>Direct Field in a Lossy Medium—Two-Dimensional Case</td>
<td>232</td>
</tr>
<tr>
<td>4.5.2</td>
<td>Direct Field in a Lossy Medium—Three-Dimensional Case</td>
<td>233</td>
</tr>
<tr>
<td>4.6</td>
<td>Finite-Difference Method</td>
<td>235</td>
</tr>
<tr>
<td>4.6.1</td>
<td>The Finite-Difference Approximation</td>
<td>236</td>
</tr>
<tr>
<td>4.6.2</td>
<td>Stability Analysis</td>
<td>239</td>
</tr>
<tr>
<td>4.6.3</td>
<td>Grid-Dispersion Error</td>
<td>242</td>
</tr>
<tr>
<td>4.6.4</td>
<td>The Yee Algorithm</td>
<td>244</td>
</tr>
<tr>
<td>4.7</td>
<td>Absorbing Boundary Conditions</td>
<td>246</td>
</tr>
<tr>
<td>4.7.1</td>
<td>Engquist-Majda Absorbing Boundary Condition</td>
<td>246</td>
</tr>
<tr>
<td>4.7.2</td>
<td>Lindman Absorbing Boundary Condition</td>
<td>249</td>
</tr>
<tr>
<td>4.7.3</td>
<td>Bayliss-Turkel Absorbing Boundary Condition</td>
<td>250</td>
</tr>
<tr>
<td>4.7.4</td>
<td>Liao’s Absorbing Boundary Condition</td>
<td>251</td>
</tr>
</tbody>
</table>
Exercises for Chapter 4 256
References for Chapter 4 262
Further Readings for Chapter 4 265

5 VARIATIONAL METHODS 271

5.1 Review of Linear Vector Space 271
5.1.1 Inner Product Spaces 271
5.1.2 Linear Operators 274
5.1.3 Basis Functions 275
5.1.4 Parseval’s Theorem 278
5.1.5 Parseval’s Theorem for Complex Vectors 279
5.1.6 Solutions to Operator Equations—a Preview 280
5.1.7 The Eigenvalue Problem 284

5.2 Variational Expressions for Self-Adjoint Problems 285
5.2.1 General Concepts 285
5.2.2 Rayleigh-Ritz Procedure—Self-Adjoint Problems 288
5.2.3 Applications to Scalar Wave Equations 291
5.2.4 Applications to Vector Wave Equations 293

5.3 Variational Expressions for Non-Self-Adjoint Problems 295
5.3.1 General Concepts 295
5.3.2 Rayleigh-Ritz Procedure—Non-Self-Adjoint Problems 297
5.3.3 Applications to Scalar Wave Equations 298
5.3.4 Applications to Vector Wave Equations 299

5.4 Variational Expressions for Eigenvalue Problems 301
5.4.1 General Concepts 301
5.4.2 Applications to Scalar Wave Equations 303
5.4.3 Applications to Electromagnetic Problems 304

5.5 Essential and Natural Boundary Conditions 308
5.5.1 The Scalar Wave Equation Case 308
5.5.2 The Electromagnetic Case 312

Exercises for Chapter 5 315
References for Chapter 5 321
Further Readings for Chapter 5 323
6  MODE MATCHING METHOD  

6.1  Eigenmodes of a Planarly Layered Medium  
   6.1.1  Orthogonality of Eigenmodes in a Layered Medium  
   6.1.2  Guided Modes and Radiation Modes of a Layered Medium  

6.2  Eigenfunction Expansion of a Field  
   6.2.1  Excitation of Modes due to a Line Source  
   6.2.2  The Use of Vector Notation  

6.3  Reflection and Transmission at a Junction Discontinuity  
   6.3.1  Derivation of Reflection and Transmission Operators  
   6.3.2  The Continuum Limit Case  

6.4  A Numerical Method to Find the Eigenmodes  

6.5  The Cylindrically Layered Medium Case  
   6.5.1  Eigenmodes of a Cylindrically Layered Medium  
   6.5.2  Differential Equations of a Cylindrical Structure  
   6.5.3  Numerical Solution of the Eigenmodes  
   6.5.4  Eigenfunction Expansion of a Field  
   6.5.5  Reflection from a Junction Discontinuity  

6.6  The Multiregion Problem  
   6.6.1  The Three-Region Problem  
   6.6.2  The N-Region Problem  

Exercises for Chapter 6  
References for Chapter 6  
Further Readings for Chapter 6  

7  DYADIC GREEN'S FUNCTIONS  

7.1  Dyadic Green's Function in a Homogeneous Medium  
   7.1.1  The Spatial Representation  
   7.1.2  The Singularity of the Dyadic Green's Function  
   7.1.3  The Spectral Representation  
   7.1.4  Equivalence of Spectral and Spatial Representations  

7.2  Vector Wave Functions  
   7.2.1  Derivation of Vector Wave Functions  
   7.2.2  Orthogonality Relationships of Vector Wave Functions  

7.2.3 Vector Wave Functions for Unbounded Media 393

7.3 Dyadic Green's Function Using Vector Wave Functions 397
7.3.1 The Integral Representations 397
7.3.2 Singularity Extraction 399

7.4 Dyadic Green's Functions for Layered Media 410
7.4.1 A General, Isotropic, Inhomogeneous Medium 410
7.4.2 Planarly Layered Media 411
7.4.3 Cylindrically Layered Media 414
7.4.4 Spherically Layered Media 416
7.4.5 Reciprocity Considerations 418

Exercises for Chapter 7 421

References for Chapter 7 424

Further Readings for Chapter 7 426

8. INTEGRAL EQUATIONS 429

8.1 Surface Integral Equations 430
8.1.1 Scalar Wave Equation 430
8.1.2 Vector Wave Equation 433
8.1.3 The Anisotropic, Inhomogeneous Medium Case 437
8.1.4 Two-Dimensional Electromagnetic Case 439

8.2 Solutions by the Method of Moments 443
8.2.1 Scalar Wave Case 443
8.2.2 The Electromagnetic Case 446
8.2.3 Problem with Internal Resonances 451

8.3 Extended-Boundary-Condition Method 453
8.3.1 The Scalar Wave Case 453
8.3.2 The Electromagnetic Wave Case 457

8.4 The Transition and Scattering Matrices 459

8.5 The Method of Rayleigh's Hypothesis 460

8.6 Scattering by Many Scatterers 463
8.6.1 Two-Scatterer Solution 463
8.6.2 $N$-Scatterer Solution—A Recursive Algorithm 465

8.7 Scattering by Multilayered Scatterers 469
8.7.1 One-Interface Problem 469
8.7.2 Many-Interface Problems 471
### 8.8 Surface Integral Equation with Finite-Element Method

<table>
<thead>
<tr>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>475</td>
</tr>
</tbody>
</table>

### 8.9 Volume Integral Equations

<table>
<thead>
<tr>
<th>Subsection</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.9.1 Scalar Wave Case</td>
<td>480</td>
</tr>
<tr>
<td>8.9.2 The Electromagnetic Wave Case</td>
<td>481</td>
</tr>
<tr>
<td>8.9.3 Matrix Representation of the Integral Equation</td>
<td>483</td>
</tr>
</tbody>
</table>

### 8.10 Approximate Solutions of the Scattering Problem

<table>
<thead>
<tr>
<th>Subsection</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.10.1 Born Approximation</td>
<td>485</td>
</tr>
<tr>
<td>8.10.2 Rytov Approximation</td>
<td>487</td>
</tr>
</tbody>
</table>

### Exercises for Chapter 8

<table>
<thead>
<tr>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>490</td>
</tr>
</tbody>
</table>

### References for Chapter 8

<table>
<thead>
<tr>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>501</td>
</tr>
</tbody>
</table>

### Further Readings for Chapter 8

<table>
<thead>
<tr>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>505</td>
</tr>
</tbody>
</table>

### 9 INVERSE SCATTERING PROBLEMS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>9.1 Linear Inverse Problems</td>
<td>511</td>
</tr>
<tr>
<td>9.1.1 Back-Projection Tomography</td>
<td>514</td>
</tr>
<tr>
<td>9.1.2 Radon Transforms</td>
<td>516</td>
</tr>
<tr>
<td>9.1.3 Diffraction Tomography</td>
<td>519</td>
</tr>
<tr>
<td>9.1.4 Finite-Source Effect</td>
<td>522</td>
</tr>
<tr>
<td>9.1.5 Nonuniqueness of the Solution</td>
<td>524</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>9.2 One-Dimensional Inverse Problems</td>
<td>526</td>
</tr>
<tr>
<td>9.2.1 The Method of Characteristics</td>
<td>526</td>
</tr>
<tr>
<td>9.2.2 Transformation to a Schrödinger-like Equation</td>
<td>532</td>
</tr>
<tr>
<td>9.2.3 The Gel’fand-Levitan Integral Equation</td>
<td>534</td>
</tr>
<tr>
<td>9.2.4 The Marchenko Integral Equation</td>
<td>541</td>
</tr>
<tr>
<td>9.2.5 The Gel’fand-Levitan-Marchenko Integral Equation</td>
<td>543</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>9.3 Higher-Dimensional Inverse Problems</td>
<td>547</td>
</tr>
<tr>
<td>9.3.1 Distorted Born Iterative Method</td>
<td>548</td>
</tr>
<tr>
<td>9.3.2 Born Iterative Method</td>
<td>553</td>
</tr>
<tr>
<td>9.3.3 Operator Forms of the Scattering Equations</td>
<td>554</td>
</tr>
</tbody>
</table>

### Exercises for Chapter 9

<table>
<thead>
<tr>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>557</td>
</tr>
</tbody>
</table>

### References for Chapter 9

<table>
<thead>
<tr>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>563</td>
</tr>
</tbody>
</table>

### Further Readings for Chapter 9

<table>
<thead>
<tr>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>566</td>
</tr>
</tbody>
</table>

### APPENDIX A Some Useful Mathematical Formulas

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>A.1 Useful Vector Identities</td>
<td>571</td>
</tr>
</tbody>
</table>