

Numerical Solution of Partial Differential Equations

An Introduction

K. W. Morton

University of Bath, UK

and

D. F. Mayers

University of Oxford, UK

Second Edition

 **CAMBRIDGE**
UNIVERSITY PRESS

Contents

<i>Preface to the first edition</i>	<i>page</i> viii
<i>Preface to the second edition</i>	xi
1 Introduction	1
2 Parabolic equations in one space variable	7
2.1 Introduction	7
2.2 A model problem	7
2.3 Series approximation	9
2.4 An explicit scheme for the model problem	10
2.5 Difference notation and truncation error	12
2.6 Convergence of the explicit scheme	16
2.7 Fourier analysis of the error	19
2.8 An implicit method	22
2.9 The Thomas algorithm	24
2.10 The weighted average or θ -method	26
2.11 A maximum principle and convergence for $\mu(1 - \theta) \leq \frac{1}{2}$	33
2.12 A three-time-level scheme	38
2.13 More general boundary conditions	39
2.14 Heat conservation properties	44
2.15 More general linear problems	46
2.16 Polar co-ordinates	52
2.17 Nonlinear problems	54
<i>Bibliographic notes</i>	56
<i>Exercises</i>	56

3	2-D and 3-D parabolic equations	62
3.1	The explicit method in a rectilinear box	62
3.2	An ADI method in two dimensions	64
3.3	ADI and LOD methods in three dimensions	70
3.4	Curved boundaries	71
3.5	Application to general parabolic problems	80
	<i>Bibliographic notes</i>	83
	<i>Exercises</i>	83
4	Hyperbolic equations in one space dimension	86
4.1	Characteristics	86
4.2	The CFL condition	89
4.3	Error analysis of the upwind scheme	94
4.4	Fourier analysis of the upwind scheme	97
4.5	The Lax–Wendroff scheme	100
4.6	The Lax–Wendroff method for conservation laws	103
4.7	Finite volume schemes	110
4.8	The box scheme	116
4.9	The leap-frog scheme	123
4.10	Hamiltonian systems and symplectic integration schemes	128
4.11	Comparison of phase and amplitude errors	135
4.12	Boundary conditions and conservation properties	139
4.13	Extensions to more space dimensions	143
	<i>Bibliographic notes</i>	146
	<i>Exercises</i>	146
5	Consistency, convergence and stability	151
5.1	Definition of the problems considered	151
5.2	The finite difference mesh and norms	152
5.3	Finite difference approximations	154
5.4	Consistency, order of accuracy and convergence	156
5.5	Stability and the Lax Equivalence Theorem	157
5.6	Calculating stability conditions	160
5.7	Practical (strict or strong) stability	166
5.8	Modified equation analysis	169
5.9	Conservation laws and the energy method of analysis	177
5.10	Summary of the theory	186
	<i>Bibliographic notes</i>	189
	<i>Exercises</i>	190

6	Linear second order elliptic equations in two dimensions	194
6.1	A model problem	194
6.2	Error analysis of the model problem	195
6.3	The general diffusion equation	197
6.4	Boundary conditions on a curved boundary	199
6.5	Error analysis using a maximum principle	203
6.6	Asymptotic error estimates	213
6.7	Variational formulation and the finite element method	218
6.8	Convection–diffusion problems	224
6.9	An example	228
	<i>Bibliographic notes</i>	231
	<i>Exercises</i>	232
7	Iterative solution of linear algebraic equations	235
7.1	Basic iterative schemes in explicit form	237
7.2	Matrix form of iteration methods and their convergence	239
7.3	Fourier analysis of convergence	244
7.4	Application to an example	248
7.5	Extensions and related iterative methods	250
7.6	The multigrid method	252
7.7	The conjugate gradient method	258
7.8	A numerical example: comparisons	261
	<i>Bibliographic notes</i>	263
	<i>Exercises</i>	263
	<i>References</i>	267
	<i>Index</i>	273