

Contents

1	Introduction	13
1.1	Examples of extremal paths in graphs	13
1.2	Basic Terminology	14
2	Path functions	21
2.1	Introduction of path functions and of related terms	21
2.2	Simple methods to define path functions	26
2.3	Order preserving path functions	31
2.3.1	Definition of order preservation and monotonicity	32
2.3.2	Structural properties of order preserving path functions	34
2.3.3	Further methods to construct order preserving functions	38
2.3.4	Modified versions of order preservation	51
2.3.5	Further remarks and definitions concerning modified order preserving principles	62
2.4	Bellman principles for paths in graphs	64
2.4.1	Introduction to strong Bellman principles for path functions	65
2.4.2	Introduction to weak Bellman principles for path functions	70
2.4.3	Comparing Bellman principles with each other	72
2.4.4	Relationships between Bellman properties and principles of order preservation	81
2.4.5	Relationships between Bellman properties and L -minimum preservation	86
2.4.6	Structural properties of optimal paths if Bellman principles are given	95
2.4.7	Functional equations and similar structural properties of cost functions with Bellman properties	99
2.4.8	Decision models with Bellman properties	101
2.5	Further definitions and results related to path functions	101
2.5.1	Prefix, infix, and suffix monotone path functions	101
2.5.2	Introduction to heuristic functions	103
2.5.3	Negative cycles	104

2.5.4	Consistent functions	107
2.5.5	Admissibility of path functions, of heuristic functions, and of evaluation functions	108
2.5.6	Triangle inequalities	109
2.5.7	Nonmisleading node functions, proper node functions and non-misleading path functions	114
2.5.8	Lexicographic path functions	115
2.5.9	Dioid properties of paths in graphs	119

3 Combinatorial results about paths in graphs **123**

3.1	Basic definitions and results	124
3.2	Short cycles and other short paths in directed and half directed graphs	129
3.3	Short cycles and other short paths in undirected graphs	131
3.3.1	Short cycles in undirected graphs	131
3.3.2	Short acyclic paths in undirected graphs	134
3.4	Definitions and results in connexion with paths of arbitrary lengths . .	143
3.5	Long cycles and long acyclic paths in digraphs	146
3.6	Long cycles and other long paths in undirected graphs	153
3.6.1	Relationships between long paths and degrees of nodes	153
3.6.2	Describing the maximal length of cycles and other paths without using the degrees of nodes	166
3.6.3	Further results about long cycles and other long paths	169

4 Algorithmic search for optimal paths in graphs and for other optimal discrete objects **171**

4.1	Introduction to optimal path problems	173
4.1.1	A simple optimal path problem and Dijkstra's algorithm	173
4.1.2	Three well-known problems and their relationships to optimal path problems	175
4.1.3	Measures for the complexity of optimal path algorithms	178
4.1.4	Formal descriptions of optimal path algorithms	179
4.2	The Single-Source-Single-Target problem	179
4.2.1	Abstract search strategies for paths in graphs	180
4.2.2	The algorithm BF**	184
4.2.3	Modifications of BF** and BF***	249
4.2.4	The algorithm GENGOAL: a search strategy whose redirection function is not order preserving	253
4.2.5	A further algorithm for extremal goal paths: MarkA	259
4.2.6	Further problems	266
4.3	The Single-Source-All-Targets problem	267
4.3.1	The algorithm FORD-BELLMAN 1	267
4.3.2	The algorithm FORD-BELLMAN 2	282
4.3.3	The algorithm FORD-BELLMAN 3	285
4.3.4	Algorithms computing minimal costs of paths	287

4.3.5	Finding C -minimal s - v -paths if $C : \mathcal{P}(s) \rightarrow (\mathbf{R}, \preceq)$ and if \preceq is neither total nor identitive	292
4.3.6	Using Best-First Search instead of Ford-Bellman strategies	293
4.3.7	Further results from literature	294
4.3.8	Finding C -maximal s - v -paths for all nodes v	295
4.4	The All-Pairs-Optimal-Path problem	296
4.4.1	Matrix algorithms simulating Ford-Bellman algorithms	299
4.4.2	Fast versions of the previous matrix algorithms	311
4.4.3	Floyd's fast search strategies	318
4.4.4	Complexity theoretical results	321
4.4.5	Further solutions of All-Pairs-Optimal-Path problems	323
4.5	Path problems and related questions in special graphs or in other special situations	324
4.5.1	Path problems arising from Geometry	324
4.5.2	Extremal paths in planar graphs	328
4.5.3	Path problems in graphs with a special structure or with a special application	330
4.6	Parallel search algorithms for optimal paths	331
4.6.1	Using computers without common memory	331
4.6.2	Using computers with a common memory	332
4.7	Searching for optimal paths in random graphs	333
4.8	Searching for almost optimal paths in graphs	340
4.8.1	Searching for paths with approximately minimal costs	341
4.8.2	Searching for the K best paths	341
4.9	Paths with extremal values $C_1(P)/C_2(P)$ where C_1, C_2 have a simple structure	344
4.9.1	(C_1/C_2) -extremal paths in acyclic digraphs	344
4.9.2	(C_1/C_2) -extremal cycles	352
4.9.3	Relationships between approximation theoretical problems and (C_1/C_2) -extremal cycles	355
4.10	Path problems with several path functions or with side constraints	369
4.10.1	Path problems with several path functions	369
4.10.2	Path problems with rational side constraints	371
4.10.3	Further path problems with side constraints	374
4.10.4	General remarks about path problems with side constraints	374
4.11	Hard problems	376
4.11.1	Path problems whose hardness is caused by the underlying cost function	376
4.11.2	The Traveling Salesman Problem and other hard path problems with side constraints	382
4.11.3	Further hard problems	390
4.12	Searching for optimal graphs and for other discrete objects similar to paths in graphs	391
4.12.1	Searching for optimal subgraphs in a given graph	391

4.12.2	Construction of optimal graphs that are not a part of a given supergraph	397
4.12.3	Further problems in connection with extremal paths in graphs	399

5 Relationships between paths in graphs and continuous curves **401**

5.1	Continuous curves: basic definitions, elementary facts, and relationships to paths in graphs	402
5.2	Additive functionals	406
5.3	Recursively linear functionals	410
5.4	Generalizations of recursive linearity	416
5.5	Functionals arising from initial value problems	419
5.6	Order preservation of functionals	427
5.7	Bellman properties of functionals	430
5.8	Functionals with many relative extrema	434
5.9	Approximation theoretical results	436

Bibliography **447**

Index **475**

Subject Index	475
Symbol Index	479