CONTENTS

PREFACE TO FIRST EDITION v
PREFACE TO SECOND EDITION vii
INTERDEPENDENCE GUIDE viii

CONTENTS ix

CHAPTER 0. PRELIMINARIES 1
1. Notation 1
2. Martingales 3
3. The monotone class theorem 4
4. Topological spaces and groups 6

CHAPTER 1. TRANSITION PROBABILITIES. MARKOV CHAINS 8
1. Kernels. Transition probabilities 8
2. Homogeneous Markov chains 13
3. Stopping times. Strong Markov property 23
4. Random walks on groups and homogeneous spaces 28
5. Analytical properties of integral kernels 33

CHAPTER 2. POTENTIAL THEORY 40
1. Superharmonic functions and the maximum principle 40
2. Reduced functions and balayage 48
3. Equilibrium, invariant events and transient sets 55
4. Invariant and excessive measures 60
5. Randomized stopping times and filling scheme 67
6. Resolvents 74

CHAPTER 3. TRANSIENCE AND RECURRENCE 80
1. Discrete Markov chains 80
2. Irreducible chains and Harris chains 87

3. Topological recurrence of random walks 98
4. Recurrence criteria for random walks and applications 106

CHAPTER 4. POINTWISE ERGODIC THEORY 117
1. Preliminaries 117
2. Maximal ergodic lemma. Hopf's decomposition 122
3. The Chacon–Ornstein theorem for conservative contractions 131
4. Applications to Harris chains 138
5. Brunel's lemma and the general Chacon–Ornstein theorem 145
6. Subadditive ergodic theory 152

CHAPTER 5. TRANSIENT RANDOM WALKS. RENEWAL THEORY 159
1. The theorem of Choquet and Deny 159
2. General lemmas 164
3. The renewal theorem for the groups **R** and **Z** 169
4. The renewal theorem 177
5. Refinements and applications 180

CHAPTER 6. ERGODIC THEORY OF HARRIS CHAINS 186
1. The zero-two laws 186
2. Cyclic classes and limit theorems for Harris chains 194
3. Quasi-compact transition probabilities and strong ergodic theorem . 201
4. Special functions 211
5. Potential kernels 216
6. The ratio-limit theorem 226

CHAPTER 7. MARTIN BOUNDARY 233
1. Regular functions 233
2. Convergence to the boundary 241
3. Integral representation of harmonic functions 251

CHAPTER 8. POTENTIAL THEORY FOR HARRIS CHAINS 261
1. Harris chains and duality 261
2. Equilibrium, balayage and maximum principles 266
3. Normal chains 271
4. Feller chains and recurrent boundary theory 277

CHAPTER 9. RECURRENT RANDOM WALKS 286
1. Preliminaries 286

2. Normality and potential kernels 298
3. Martin boundary 308
4. Renewal theory. 315

CHAPTER 10. CONSTRUCTION OF MARKOV CHAINS AND RESOLVENTS 324
1. Preliminaries and bounded kernels 324
2. The reinforced principle. Construction of transient Markov chains . . 330
3. The semi-complete maximum principle 338

NOTES AND COMMENTS 346

REFERENCES 356
INDEX OF NOTATION 371
INDEX OF TERMS 372