CONTENTS

PART A : "LIMIT THEOREMS FOR PRODUCTS OF RANDOM MATRICES"

INTRODUCTION 1

CHAPTER I - THE UPPER LYAPUNOV EXPONENT 5

1. Notation 5
2. The upper Lyapunov exponent 6
3. Cocycles 8
4. The theorem of Furstenberg and Kesten . . . 11
5. Exercises 13

CHAPTER II - MATRICES OF ORDER TWO 17

1. The set-up 17
2. Two basic lemmas 19
3. Contraction properties 24
4. Furstenberg's theorem 30
5. Some simple examples 33
6. Exercises 36
7. Complements 38

CHAPTER III - CONTRACTION PROPERTIES 43

1. Contracting sets 44
2. Strong irreducibility 48
3. A key property 50
4. Contracting action on $P(\mathbb{R}^d)$ and convergence in direction 55

5. Lyapunov exponents 60

6. Comparison of the top Lyapunov exponents and Furstenberg's theorem 64

7. Complements. The irreducible case 68

CHAPTER IV - COMPARISON OF LYAPUNOV EXPONENTS AND BOUNDARIES 77

1. A criterion ensuring that Lyapunov exponents are distinct 77

2. Some examples 81

3. The case of symplectic matrices 87

4. μ-boundaries 93

CHAPTER V - CENTRAL LIMIT THEOREMS AND RELATED RESULTS . 101

1. Introduction 101

2. Exponential convergence to the invariant measure 103

3. A lemma of perturbation theory 111

4. The Fourier-Laplace transform near 0 116

5. Central limit theorem 121

6. Large deviations 129

7. Convergence to γ_p 134

8. Convergence in distribution without normalization 135

9. Complements : linear stochastic differential equations 140

CHAPTER VI - PROPERTIES OF THE INVARIANT MEASURE AND APPLICATIONS 145

1. Convergence in the Iwasawa decomposition . . 147

2. Limit theorems for the coefficients 155

3. Behaviour of the rows 159

4. Regularity of the invariant measure 161

5. An example : random continued fractions . . 166

SUGGESTIONS FOR FURTHER READINGS 173

BIBLIOGRAPHY 175

PART B : "RANDOM SCHRÖDINGER OPERATORS"

INTRODUCTION 183

CHAPTER I - THE DETERMINISTIC SCHRÖDINGER OPERATOR . . 187

1. The difference equation. Hyperbolic structures . 187
2. Self adjointness of H. Spectral properties . . 190
3. Slowly increasing generalized eigenfunctions . 195
4. Approximations of the spectral measure . . . 196
5. The pure point spectrum. A criterion . . . 200
6. Singularity of the spectrum 202

CHAPTER II - ERGODIC SCHRÖDINGER OPERATORS 205

1. Definition and examples 205
2. General spectral properties 206
3. The Lyapunov exponent in the general ergodic case 209
4. The Lyapunov exponent in the independent case . 211
5. Absence of absolutely continuous spectrum . . 221
6. Distribution of states. Thouless formula . . 224
7. The pure point spectrum. Kotani's criterion . 232
8. Asymptotic properties of the conductance in the disordered wire 234

CHAPTER III - THE PURE POINT SPECTRUM 237

1. The pure point spectrum. First proof . . . 238
2. The Laplace transform on $Sl(2,\mathbb{R})$ 240
3. The pure point spectrum. Second proof . . . 247
4. The density of states 250

CHAPTER IV - SCHRÖDINGER OPERATORS IN A STRIP . . . 253

1. The deterministic Schrödinger operator in a strip 253
2. Ergodic Schrödinger operators in a strip . . 259
3. Lyapunov exponents in the independent case. The pure point spectrum (first proof) . . . 262
4. The Laplace transform on $Sp(\ell,\mathbb{R})$ 267
5. The pure point spectrum, second proof . . . 272

APPENDIX 275

BIBLIOGRAPHY 277