

Contents

Chapter I

The prime number theorem and Selberg's method

§ 1. Selberg's formula	1
§ 2. A variant of Selberg's formula	6
§ 3. Wirsing's inequality	12
§ 4. The prime number theorem	17
§ 5. The order of magnitude of the divisor function	19
Notes on Chapter I	21

Chapter II

The zeta-function of Riemann

§ 1. The functional equation	28
§ 2. The Riemann-von Mangoldt formula	33
§ 3. The entire function ξ	40
§ 4. Hardy's theorem	45
§ 5. Hamburger's theorem	51
Notes on Chapter II	54

Chapter III

Littlewood's theorem and Weyl's method

§ 1. Zero-free region of ζ	58
§ 2. Weyl's inequality	60
§ 3. Some results of Hardy and Littlewood and of Weyl	69
§ 4. Littlewood's theorem	73
§ 5. Applications of Littlewood's theorem	78
Notes on Chapter III	84

Chapter IV

Vinogradov's method

§ 1. A refinement of Littlewood's theorem	88
§ 2. An outline of the method	88
§ 3. Vinogradov's mean-value theorem	90
§ 4. Vinogradov's inequality	99
§ 5. Estimation of sections of $\zeta(s)$ in the critical strip	106

§ 6. Chudakov's theorem	108
§ 7. Approximation of $\pi(x)$	110
Notes on Chapter IV	110

Chapter V

Theorems of Hoheisel and of Ingham

§ 1. The difference between consecutive primes	112
§ 2. Landau's formula for the Chebyshev function ψ	113
§ 3. Hoheisel's theorem	124
§ 4. Two auxiliary lemmas	126
§ 5. Ingham's theorem	130
§ 6. An application of Chudakov's theorem	138
Notes on Chapter V	139

Chapter VI

Dirichlet's L -functions and Siegel's theorem

§ 1. Characters and L -functions	143
§ 2. Zeros of L -functions	145
§ 3. Proper characters	146
§ 4. The functional equation of $L(s, \chi)$	149
§ 5. Siegel's theorem	155
Notes on Chapter VI	164

Chapter VII

Theorems of Hardy-Ramanujan and of Rademacher on the partition function

§ 1. The partition function	166
§ 2. A simple case	166
§ 3. A bound for $p(n)$	169
§ 4. A property of the generating function of $p(n)$	170
§ 5. The Dedekind η -function	174
§ 6. The Hardy-Ramanujan formula	178
§ 7. Rademacher's identity	185
Notes on Chapter VII	191

Chapter VIII

Dirichlet's divisor problem

§ 1. The average order of the divisor function	194
§ 2. An application of Perron's formula	195
§ 3. An auxiliary function	198
§ 4. An identity involving the divisor function	200
§ 5. Voronoi's theorem	202

§ 6. A theorem of A. S. Besicovitch	204
§ 7. Theorems of Hardy and of Ingham	205
§ 8. Equiconvergence theorems of A. Zygmund	209
§ 9. The Voronoi identity	223
Notes on Chapter VIII	226
A list of books	229
Subject index	230