

The Royal Mathematician was a bald-bearded, nearsighted man, with a skullcap on his head and a pencil behind each ear. He wore a black suit with white numbers on it.

"I don't want to hear a long list of all the things you have figured out for me since 1907," the King said to him. "I just want you to figure out right now how to get the moon for the Princess Lenore. When she gets the moon, she will be well again."

"I am glad you mentioned all the things I have figured out for you since 1907," said the Royal Mathematician. "It so happens that I have a list of them with me."

James Thurber, "Many Moons"

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