

**Mathematical  
Surveys  
and  
Monographs**  
Volume 120

# Trace Ideals and Their Applications

## Second Edition

**Barry Simon**



**American Mathematical Society**

# Contents

Preface to the Second Edition	v
Preface to the First Edition	vii
Chapter 1. Preliminaries	1
1.1. Absolute Value and Polar Decomposition	1
1.2. Compact Operators and the Canonical Decomposition	1
1.3. Inequalities on Singular Values, I	3
1.4. Rearrangement Inequalities and All That	4
1.5. Antisymmetric Tensor Products	6
1.6. Inequalities on Eigenvalues, I	8
1.7. Symmetrically Normed Spaces	8
1.8. Inequalities on Singular Values and Eigenvalues, II	11
1.9. Clarkson-McCarthy Inequalities	14
Chapter 2. Calkin's Theory of Operator Ideals and Symmetrically Normed Ideals; Convergence Theorems for $\mathcal{J}_P$	17
Chapter 3. Trace, Determinant, and Lidskii's Theorem	31
Chapter 4. $f(x)g(-i\nabla)$	37
Chapter 5. Fredholm Theory	45
Chapter 6. Scattering With a Trace Condition	53
Chapter 7. Bound State Problems	61
Chapter 8. Lots of Inequalities	67
8.1. Golden-Thompson Inequalities	67
8.2. Lieb's Inequalities	70
8.3. Peierls-Bogoliubov and Berezin Inequalities	71
8.4. Lieb Concavity	72
Chapter 9. Regularized Determinants and Renormalization in Quantum Field Theory	75
Chapter 10. An Introduction to the Theory on a Banach Space	81
Chapter 11. Borel Transforms, the Krein Spectral Shift, and All That	85
11.1. Borel Transforms of Measures	86
11.2. Rank One Perturbations: The Set-Up and Basic Formulae	90

11.3. The Integral Formula	92
11.4. The Krein Spectral Shift	93
Appendix to Section 11.4: The Krein Spectral Shift for Trace Class Perturbations	96
11.5. Infinite Coupling	97
11.6. Boundary Condition Variation of ODE's	100
11.7. Jacobi Matrices	102
Chapter 12. Spectral Theory of Rank One Perturbations	105
12.1. Invariance of the Absolutely Continuous Spectrum	105
12.2. The Aronszajn-Donoghue Theory	106
12.3. The Simon-Wolff Criterion	107
12.4. Instability of Point Spectrum	109
12.5. Examples	110
Chapter 13. Localization in the Anderson Model Following Aizenman-Molchanov	113
Chapter 14. The Xi Function	117
14.1. Abstract Trace Formula	118
14.2. The Trace Formula for Schrödinger Operators	119
14.3. Examples	120
14.4. The Trace Formula for Jacobi Matrices	122
14.5. A Regularity Theorem for the A.C. Spectrum	123
14.6. Inverse Problems	124
Appendix. Addenda	127
Bibliography	135
Index	149