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Redefining Geometrical Exactness

Descartes' Transformation of the Early
Modern Concept of Construction

With 95 Illustrations



Springer

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