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Boundary-Value Problems for Gravimetric Determination of a Precise Geoid

With 51 Figures and 3 Tables



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