

Introductory Statistics and Random Phenomena

Uncertainty, Complexity and
Chaotic Behavior in
Engineering and Science

with *Mathematica*® *Uncertain Virtual Worlds*™
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Boston • Basel • Berlin

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