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# From Instability to Intelligence

Complexity and Predictability  
in Nonlinear Dynamics



Springer

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