

Pol Vanhaecke

Integrable Systems in the realm of Algebraic Geometry



Springer

Table of Contents

| | |
|---|-----------|
| I. Introduction | 1 |
| II. Integrable Hamiltonian systems on affine Poisson varieties | 17 |
| 1. Introduction | 17 |
| 2. Affine Poisson varieties and their morphisms | 19 |
| 2.1. Affine Poisson varieties | 19 |
| 2.2. Morphisms of affine Poisson varieties | 25 |
| 2.3. Constructions of affine Poisson varieties | 27 |
| 2.4. Decompositions and invariants of affine Poisson varieties | 33 |
| 3. Integrable Hamiltonian systems and their morphisms | 43 |
| 3.1. Integrable Hamiltonian systems on affine Poisson varieties | 43 |
| 3.2. Morphisms of integrable Hamiltonian systems | 50 |
| 3.3. Constructions of integrable Hamiltonian systems | 53 |
| 3.4. Compatible integrable Hamiltonian systems and multi-Hamiltonian systems | 57 |
| 4. Integrable Hamiltonian systems on other spaces | 60 |
| 4.1. Poisson spaces | 60 |
| 4.2. Integrable Hamiltonian systems on Poisson spaces | 64 |
| III. Integrable Hamiltonian systems and symmetric products of curves | 67 |
| 1. Introduction | 67 |
| 2. The systems and their integrability | 69 |
| 2.1. Notation | 69 |
| 2.2. The compatible Poisson structures $\{\cdot, \cdot\}_d^\varphi$ | 69 |
| 2.3. Polynomials in involution for $\{\cdot, \cdot\}_d^\varphi$ | 74 |
| 2.4. The hyperelliptic case | 79 |
| 3. The geometry of the level manifolds | 82 |
| 3.1. The level sets $\mathcal{F}_{F,d}^{\mathbf{R}}$ and $\mathcal{F}_{F,d}$ | 82 |
| 3.2. The structure of the complex level manifolds $\mathcal{F}_{F,d}$ | 83 |
| 3.3. The structure of the real level manifolds $\mathcal{F}_{F,d}^{\mathbf{R}}$ | 86 |
| 3.4. Compactification of the complex level manifolds $\mathcal{F}_{F,d}$ | 90 |
| 3.5. The significance of the Poisson structures $\{\cdot, \cdot\}_d^\varphi$ | 92 |

| | |
|---|------------|
| IV. Interludium: the geometry of Abelian varieties | 95 |
| 1. Introduction | 95 |
| 2. Divisors and line bundles | 97 |
| 2.1. Divisors | 97 |
| 2.2. Line bundles | 98 |
| 2.3. Sections of line bundles | 99 |
| 2.4. The Riemann-Roch Theorem | 101 |
| 2.5. Line bundles and embeddings in projective space | 103 |
| 2.6. Hyperelliptic curves | 104 |
| 3. Abelian varieties | 106 |
| 3.1. Complex tori and Abelian varieties | 106 |
| 3.2. Line bundles on Abelian varieties | 107 |
| 3.3. Abelian surfaces | 109 |
| 4. Jacobi varieties | 112 |
| 4.1. The algebraic Jacobian | 112 |
| 4.2. The analytic/trancendental Jacobian | 112 |
| 4.3. Abel's Theorem and Jacobi inversion | 116 |
| 4.4. Jacobi and Kummer surfaces | 118 |
| 4.5. Abelian surfaces of type (1,4) | 120 |
| V. Algebraic completely integrable Hamiltonian systems | 123 |
| 1. Introduction | 123 |
| 2. A.c.i. systems | 125 |
| 3. Painlevé analysis for a.c.i. systems | 131 |
| 4. Linearization of two-dimensional a.c.i. systems | 134 |
| 5. Lax equations | 136 |
| VI. The master systems | 139 |
| 1. Introduction | 139 |
| 2. The odd and the even master systems | 141 |
| 2.1. Algebraic complete integrability | 141 |
| 2.2. A multi-Hamiltonian structure for the master systems | 146 |
| 2.3. The Bechlivanidis-van Moerbeke system | 150 |
| 3. Application: generalized Kummer surfaces | 154 |
| 3.1. An equation for the curve Γ | 154 |
| 3.2. The 9_4 configuration on the Jacobian of Γ | 156 |
| 3.3. Equations for $J(\Gamma)/\tau$ in \mathbf{P}^4 | 159 |
| 4. The general case | 165 |

| | |
|---|------------|
| VII. The Garnier and Hénon-Heiles potentials and the Toda lattice . . . | 171 |
| 1. Introduction | 171 |
| 2. The Garnier potential, Abelian surfaces of type (1,4) and their moduli | 173 |
| 2.1. The quartic potential $V_{\alpha\beta}$ and its integrability | 173 |
| 2.2. Some moduli spaces of Abelian surfaces of type (1,4) | 179 |
| 2.3. The precise relation with the canonical Jacobian | 183 |
| 2.4. The relation with the canonical Jacobian made explicit | 188 |
| 2.5. The central potentials $V_{\alpha\alpha}$ | 193 |
| 3. The Hénon-Heiles hierarchy | 197 |
| 3.1. The cubic Hénon-Heiles potential | 197 |
| 3.2. The quartic Hénon-Heiles potential | 199 |
| 3.3. The Hénon-Heiles hierarchy | 199 |
| 4. The Toda lattice | 202 |
| 4.1. Different forms of the Toda lattice | 202 |
| 4.2. A morphism to the even master system | 204 |
| 4.3. Toda and Abelian surfaces of type (1,3) | 206 |
| References | 209 |
| Index | 217 |