

Aad W. van der Vaart Jon A. Wellner

Weak Convergence and Empirical Processes

With Applications to Statistics



Springer

Contents

Preface	vii
Reading Guide	xi
1. Stochastic Convergence	1
1.1. Introduction	2
1.2. Outer Integrals and Measurable Majorants	6
1.3. Weak Convergence	16
1.4. Product Spaces	29
1.5. Spaces of Bounded Functions	34
1.6. Spaces of Locally Bounded Functions	43
1.7. The Ball Sigma-Field and Measurability of Suprema	45
1.8. Hilbert Spaces	49
1.9. Convergence: Almost Surely and in Probability	52
1.10. Convergence: Weak, Almost Uniform, and in Probability	57
1.11. Refinements	67
1.12. Uniformity and Metrization	71
<i>Notes</i>	75

2. Empirical Processes	79
2.1. Introduction	80
2.1.1. Overview of Chapters 2.3–2.14	83
2.1.2. Asymptotic Equicontinuity	89
2.1.3. Maximal Inequalities	90
*2.1.4. The Central Limit Theorem in Banach Spaces	91
2.2. Maximal Inequalities and Covering Numbers	95
2.2.1. Sub-Gaussian Inequalities	100
2.2.2. Bernstein's Inequality	102
*2.2.3. Tightness Under an Increment Bound	104
2.3. Symmetrization and Measurability	107
2.3.1. Symmetrization	107
*2.3.2. More Symmetrization	111
*2.3.3. Separable Versions	115
2.4. Glivenko-Cantelli Theorems	122
2.5. Donsker Theorems	127
2.5.1. Uniform Entropy	127
2.5.2. Bracketing	129
2.6. Uniform Entropy Numbers	134
2.6.1. VC-Classes of Sets	134
2.6.2. VC-Classes of Functions	140
2.6.3. Convex Hulls and VC-Hull Classes	142
2.6.4. VC-Major Classes	145
2.6.5. Examples and Permanence Properties	146
2.7. Bracketing Numbers	154
2.7.1. Smooth Functions and Sets	154
2.7.2. Monotone Functions	159
2.7.3. Closed Convex Sets and Convex Functions	162
2.7.4. Classes That Are Lipschitz in a Parameter	164
2.8. Uniformity in the Underlying Distribution	166
2.8.1. Glivenko-Cantelli Theorems	166
2.8.2. Donsker Theorems	168
2.8.3. Central Limit Theorem Under Sequences	173
2.9. Multiplier Central Limit Theorems	176
2.10. Permanence of the Donsker Property	190
2.10.1. Closures and Convex Hulls	190
2.10.2. Lipschitz Transformations	192
2.10.3. Permanence of the Uniform Entropy Bound	198
2.10.4. Partitions of the Sample Space	200
2.11. The Central Limit Theorem for Processes	205
2.11.1. Random Entropy	205
2.11.2. Bracketing	210
2.11.3. Classes of Functions Changing with n	220
2.12. Partial-Sum Processes	225

2.12.1. The Sequential Empirical Process	225
2.12.2. Partial-Sum Processes on Lattices	228
2.13. Other Donsker Classes	232
2.13.1. Sequences	232
2.13.2. Elliptical Classes	233
2.13.3. Classes of Sets	236
2.14. Tail Bounds	238
2.14.1. Finite Entropy Integrals	238
2.14.2. Uniformly Bounded Classes	245
2.14.3. Deviations from the Mean	254
2.14.4. Proof of Theorem 2.14.13	257
<i>Notes</i>	269
3. Statistical Applications	277
3.1. Introduction	278
3.2. M-Estimators	284
3.2.1. The Argmax Theorem	285
3.2.2. Rate of Convergence	289
3.2.3. Examples	294
3.2.4. Linearization	300
3.3. Z-Estimators	309
3.4. Rates of Convergence	321
3.4.1. Maximum Likelihood	326
3.4.2. Concave Parametrizations	330
3.4.3. Least Squares Regression	331
3.4.4. Least-Absolute-Deviation Regression	336
3.5. Random Sample Size, Poissonization and Kac Processes	339
3.5.1. Random Sample Size	339
3.5.2. Poissonization	341
3.6. The Bootstrap	345
3.6.1. The Empirical Bootstrap	345
3.6.2. The Exchangeable Bootstrap	353
3.7. The Two-Sample Problem	360
3.7.1. Permutation Empirical Processes	362
3.7.2. Two-Sample Bootstrap	365
3.8. Independence Empirical Processes	367
3.9. The Delta-Method	372
3.9.1. Main Result	372
3.9.2. Gaussian Limits	376
3.9.3. The Delta-Method for the Bootstrap	377
3.9.4. Examples of the Delta-Method	381
3.10. Contiguity	401
3.10.1. The Empirical Process	406

3.10.2. Change-Point Alternatives	408
3.11. Convolution and Minimax Theorems	412
3.11.1. Efficiency of the Empirical Distribution	420
Notes	423
A. Appendix	429
A.1. Inequalities	430
A.2. Gaussian Processes	437
A.2.1. Inequalities and Gaussian Comparison	437
A.2.2. Exponential Bounds	442
A.2.3. Majorizing Measures	445
A.2.4. Further Results	447
A.3. Rademacher Processes	449
A.4. Isoperimetric Inequalities for Product Measures	451
A.5. Some Limit Theorems	456
A.6. More Inequalities	459
A.6.1. Binomial Random Variables	459
A.6.2. Multinomial Random Vectors	462
A.6.3. Rademacher Sums	463
Notes	465
References	467
Author Index	487
Subject Index	493
List of Symbols	506