

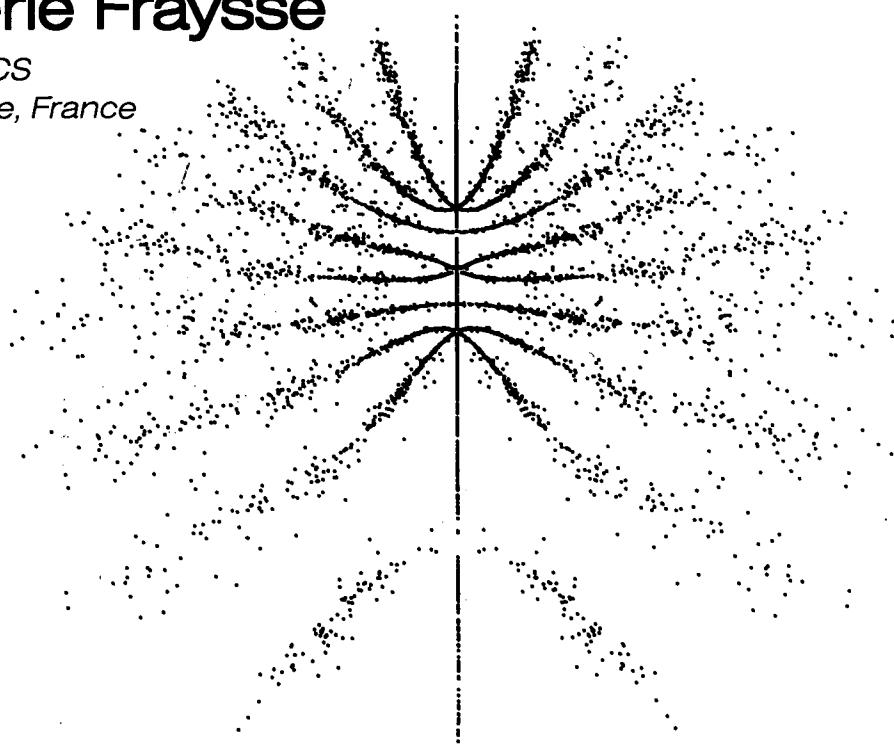
# Lectures *on* Finite Precision Computations

**Françoise Chaitin-Chatelin**

*University Paris IX Dauphine, CEREMADE and CERFACS  
Toulouse, France*

**Valérie Frayssé**

*CERFACS  
Toulouse, France*



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Society for Industrial and Applied Mathematics  
Philadelphia

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