Identical Parallel Machine Scheduling Problems: 
Structural patterns, bounding techniques and 
solution procedures

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