

VRIJE UNIVERSITEIT

Rare Event Simulation Techniques for Stochastic Design  
Problems in Markovian Setting

ACADEMISCH PROEFSCHRIFT

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