Modelling Under Risk and Uncertainty

An Introduction to Statistical, Phenomenological and Computational Methods

Etienne de Rocquigny

Ecole Centrale Paris, Université Paris-Saclay, France

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