Strategic Financial Planning over the Lifecycle

A Conceptual Approach to Personal Risk Management

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