Financial Modeling, Actuarial Valuation and Solvency in Insurance
Contents

1 Introduction .............................................. 1
  1.1 Full Balance Sheet Approach ......................... 3
  1.2 Solvency Considerations .............................. 4
  1.3 Further Modeling Issues ............................... 5
  1.4 Outline of This Book .................................. 6

Part I  Financial Valuation Principles

2 State Price Deflators and Stochastic Discounting ................. 11
  2.1 Zero Coupon Bonds and Term Structure of Interest Rates ......... 11
    2.1.1 Motivation for Discounting ....................... 11
    2.1.2 Spot Rates and Term Structure of Interest Rates ...... 12
    2.1.3 Estimating the Yield Curve ......................... 15
  2.2 Basic Discrete Time Stochastic Model .................... 18
    2.2.1 Valuation at Time 0 ................................ 19
    2.2.2 Interpretation of State Price Deflators .............. 22
    2.2.3 Valuation at Time $t > 0$ .......................... 23
  2.3 Equivalent Martingale Measure .......................... 26
    2.3.1 Bank Account Numeraire ............................ 26
    2.3.2 Martingale Measure and the FTAP .................... 27
  2.4 Market Price of Risk .................................. 31

3 Spot Rate Models ........................................... 35
  3.1 General Gaussian Spot Rate Models ..................... 35
  3.2 One-Factor Gaussian Affine Term Structure Models ............ 38
  3.3 Discrete Time One-Factor Vasicek Model .................. 41
    3.3.1 Spot Rate Dynamics on a Yearly Grid ................. 42
    3.3.2 Spot Rate Dynamics on a Monthly Grid ............... 45
    3.3.3 Parameter Calibration in the One-Factor Vasicek Model . 47
  3.4 Conditionally Heteroscedastic Spot Rate Models .............. 56
  3.5 Auto-Regressive Moving Average (ARMA) Spot Rate Models ....... 60
    3.5.1 AR(1) Spot Rate Model ............................ 61
### Contents

3.5.2 AR(p) Spot Rate Model ................................................. 62
3.5.3 General ARMA Spot Rate Models ................................. 63
3.5.4 Parameter Calibration in ARMA Models ......................... 64
3.6 Discrete Time Multifactor Vasicek Model ......................... 65
3.6.1 Motivation for Multifactor Spot Rate Models .................... 65
3.6.2 Multifactor Vasicek Model (with Independent Factors) ....... 67
3.6.3 Parameter Estimation and the Kalman Filter .................... 72
3.7 One-Factor Gamma Spot Rate Model ............................... 87
3.7.1 Gamma Affine Term Structure Model ............................ 87
3.7.2 Parameter Calibration in the Gamma Spot Rate Model .......... 90
3.8 Discrete Time Black–Karasinski Model .............................. 92
3.8.1 Log-Normal Spot Rate Dynamics .................................. 92
3.8.2 Parameter Calibration in the Black–Karasinski Model ......... 93
3.8.3 ARMA Extended Black–Karasinski Model ....................... 95

4 **Stochastic Forward Rate and Yield Curve Modeling** .................. 97
4.1 General Discrete Time HJM Framework .............................. 98
4.2 Gaussian Discrete Time HJM Framework ............................ 100
4.2.1 General Gaussian Discrete Time HJM Framework ............... 100
4.2.2 Two-Factor Gaussian HJM Model ................................. 102
4.2.3 Nelson–Siegel and Svensson HJM Framework .................... 105
4.3 Yield Curve Modeling .................................................. 106
4.3.1 Derivations from the Forward Rate Framework ................. 106
4.3.2 Stochastic Yield Curve Modeling .................................. 109

Appendix Proofs of Chap. 4 ................................................. 125

5 **Pricing of Financial Assets** ............................................. 131
5.1 Pricing of Cash Flows .................................................. 132
5.1.1 General Cash Flow Valuation in the Vasicek Model .......... 132
5.1.2 Defaultable Coupon Bonds .......................................... 135
5.2 Financial Market .......................................................... 137
5.2.1 A Log-Normal Example in the Vasicek Model ................... 139
5.2.2 A First Asset-and-Liability Management Problem .............. 143
5.3 Pricing of Derivative Instruments .................................... 146

Appendix Proofs of Chap. 5 ................................................. 149

**Part II**  
**Actuarial Valuation and Solvency** ........................................ 155
6 **Actuarial and Financial Modeling** ...................................... 155
6.1 Financial Market and Financial Filtration .......................... 155
6.2 Basic Actuarial Model .................................................. 157
6.3 Improved Actuarial Model .............................................. 164

7 **Valuation Portfolio** .................................................... 169
7.1 Construction of the Valuation Portfolio ............................. 170
7.1.1 Financial Portfolios and Cash Flows ............................ 171
7.1.2 Construction of the VaPo ............................................ 171
8 Protected Valuation Portfolio

8.1 Construction of the Protected Valuation Portfolio

8.2 Market-Value Margin

8.2.1 Risk-Adjusted Reserves

8.2.2 Claims Development Result of Risk-Adjusted Reserves

8.2.3 Fortuin–Kasteleyn–Ginibre (FKG) Inequality

8.2.4 Examples in Life Insurance

8.2.5 Example in Non-life Insurance

8.2.6 Further Probability Distortion Examples

8.3 Numerical Examples

8.3.1 Non-life Insurance Run-Off

8.3.2 Life Insurance Examples

9 Solvency

9.1 Risk Measures

9.1.1 Definition of (Conditional) Risk Measures

9.1.2 Examples of Risk Measures

9.2 Solvency and Acceptability

9.2.1 Definition of Solvency and Acceptability

9.2.2 Free Capital and Solvency Terminology

9.2.3 Insolvency

9.3 No Insurance Technical Risk

9.3.1 Theoretical ALM Solution and Free Capital

9.3.2 General Asset Allocations

9.3.3 Limited Liability Option

9.3.4 Margrabe Option

9.3.5 Hedging Margrabe Options

9.4 Inclusion of Insurance Technical Risk

9.4.1 Insurance Technical and Financial Result

9.4.2 Theoretical ALM Solution and Solvency

9.4.3 General ALM Problem and Insurance Technical Risk

9.4.4 Cost-of-Capital Loading and Dividend Payments

9.4.5 Risk Spreading and Law of Large Numbers

9.4.6 Limitations of the Vasicek Financial Model

9.5 Portfolio Optimization

9.5.1 Standard Deviation Based Risk Measure

9.5.2 Estimation of the Covariance Matrix
10 Selected Topics and Examples .................................. 337
  10.1 Extreme Value Distributions and Copulas ............... 337
  10.2 Parameter Uncertainty ....................................... 339
    10.2.1 Parameter Uncertainty for a Non-life Run-Off .......... 339
    10.2.2 Modeling of Longevity Risk ............................. 352
  10.3 Cost-of-Capital Loading in Practice ....................... 356
    10.3.1 General Considerations ................................. 356
    10.3.2 Cost-of-Capital Loading Example ....................... 358
  10.4 Accounting Year Factors in Run-Off Triangles ............. 366
    10.4.1 Model Assumptions ..................................... 366
    10.4.2 Predictive Distribution ................................ 368
  10.5 Premium Liability Modeling .................................. 369
    10.5.1 Modeling Attritional Claims ............................ 371
    10.5.2 Modeling Large Claims ................................ 375
    10.5.3 Reinsurance ........................................... 376
  10.6 Risk Measurement and Solvency Modeling ................... 381
    10.6.1 Insurance Liabilities .................................. 381
    10.6.2 Asset Portfolio and Premium Income .................... 385
    10.6.3 Cost Process and Other Risk Factors ................... 387
    10.6.4 Accounting Condition and Acceptability ............... 388
    10.6.5 Solvency Toy Model in Action ......................... 390
  10.7 Concluding Remarks ......................................... 402

Part III Appendix

11 Auxiliary Considerations .................................... 407
  11.1 Helpful Results with Gaussian Distributions ............. 407
  11.2 Change of Numeraire Technique .............................. 408
    11.2.1 General Changes of Numeraire .......................... 408
    11.2.2 Forward Measures and European Options on ZCBs .......... 410
    11.2.3 European Options with Log-Normal Asset Prices ....... 415

References ...................................................... 419

Index .......................................................... 427