Contents

List of Figures xiii
List of Tables xv

1 Introduction and Overview 1
1.1 What is fixed income analysis? 1
1.2 Basic bond market terminology 2
1.3 Bond markets and money markets 12
1.4 Fixed income derivatives 18
1.5 An overview of the book 21
1.6 Exercises 23

2 Extracting Yield Curves from Bond Prices 25
2.1 Introduction 25
2.2 Bootstrapping 26
2.3 Cubic splines 29
2.4 The Nelson–Siegel parametrization 33
2.5 Additional remarks on yield curve estimation 36
2.6 Exercises 37

3 Stochastic Processes and Stochastic Calculus 38
3.1 Introduction 38
3.2 What is a stochastic process? 39
3.3 Brownian motions 47
3.4 Diffusion processes 52
3.5 Itô processes 53
3.6 Stochastic integrals 54
3.7 Itô’s Lemma 58
3.8 Important diffusion processes 59
3.9 Multi-dimensional processes 68
3.10 Change of probability measure 76
3.11 Exercises 80

4 A Review of General Asset Pricing Theory 82
4.1 Introduction 82
4.2 Assets, trading strategies, and arbitrage 84
4.3 State-price deflators, risk-neutral probabilities, and market prices of risk 88
4.4 Other useful probability measures 97
4.5 Complete vs. incomplete markets 100
4.6 Equilibrium and representative agents in complete markets 102
4.7 Extension to intermediate dividends 104
4.8 Diffusion models and the fundamental partial differential equation 106
4.9 Concluding remarks 114
4.10 Exercises 115
5 The Economics of the Term Structure of Interest Rates 116
5.1 Introduction 116
5.2 Real interest rates and aggregate consumption 117
5.3 Real interest rates and aggregate production 122
5.4 Equilibrium term structure models 125
5.5 Real and nominal interest rates and term structures 128
5.6 The expectation hypothesis 140
5.7 Liquidity preference, market segmentation, and preferred habitats 145
5.8 Concluding remarks 146
5.9 Exercises 147

6 Fixed Income Securities 150
6.1 Introduction 150
6.2 Forwards and futures 151
6.3 European options 158
6.4 Caps, floors, and collars 163
6.5 Swaps and swaptions 169
6.6 American-style derivatives 178
6.7 An overview of term structure models 180
6.8 Exercises 182

7 One-Factor Diffusion Models 184
7.1 Introduction 184
7.2 Affine models 185
7.3 Merton's model 196
7.4 Vasicek's model 200
7.5 The Cox–Ingersoll–Ross model 214
7.6 Generalized affine models 220
7.7 Non-affine models 221
7.8 Parameter estimation and empirical tests 224
7.9 Concluding remarks 228
7.10 Exercises 229

8 Multi-Factor Diffusion Models 231
8.1 Introduction 231
8.2 The general multi-factor setting 234
8.3 Affine multi-factor models 236
8.4 Two-factor affine diffusion models 243
8.5 Three-factor affine models 252
8.6 Generalized affine models 255
8.7 Other multi-factor diffusion models 257
8.8 Final remarks 261
8.9 Exercises 262

9 Calibration of Diffusion Models 264
9.1 Introduction 264
9.2 Time-inhomogeneous affine models 265
9.3 The Ho–Lee model (extended Merton) 267
9.4 The Hull–White model (extended Vasicek) 269
<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>9.5</td>
<td>The extended CIR model</td>
<td>273</td>
</tr>
<tr>
<td>9.6</td>
<td>Calibration to other market data</td>
<td>274</td>
</tr>
<tr>
<td>9.7</td>
<td>Initial and future term structures in calibrated models</td>
<td>275</td>
</tr>
<tr>
<td>9.8</td>
<td>Calibrated non-affine models</td>
<td>277</td>
</tr>
<tr>
<td>9.9</td>
<td>Is a calibrated one-factor model just as good as a multi-factor model?</td>
<td>278</td>
</tr>
<tr>
<td>9.10</td>
<td>Final remarks</td>
<td>280</td>
</tr>
<tr>
<td>9.11</td>
<td>Exercises</td>
<td>280</td>
</tr>
<tr>
<td>10.1</td>
<td>Introduction</td>
<td>281</td>
</tr>
<tr>
<td>10.2</td>
<td>Basic assumptions</td>
<td>281</td>
</tr>
<tr>
<td>10.3</td>
<td>Bond price dynamics and the drift restriction</td>
<td>283</td>
</tr>
<tr>
<td>10.4</td>
<td>Three well-known special cases</td>
<td>285</td>
</tr>
<tr>
<td>10.5</td>
<td>Gaussian HJM models</td>
<td>289</td>
</tr>
<tr>
<td>10.6</td>
<td>Diffusion representations of HJM models</td>
<td>292</td>
</tr>
<tr>
<td>10.7</td>
<td>HJM models with forward-rate dependent volatilities</td>
<td>298</td>
</tr>
<tr>
<td>10.8</td>
<td>HJM models with unspanned stochastic volatility</td>
<td>299</td>
</tr>
<tr>
<td>10.9</td>
<td>Concluding remarks</td>
<td>299</td>
</tr>
<tr>
<td>10.10</td>
<td>Exercises</td>
<td>300</td>
</tr>
<tr>
<td>11.1</td>
<td>Introduction</td>
<td>303</td>
</tr>
<tr>
<td>11.2</td>
<td>General LIBOR market models</td>
<td>304</td>
</tr>
<tr>
<td>11.3</td>
<td>The lognormal LIBOR market model</td>
<td>312</td>
</tr>
<tr>
<td>11.4</td>
<td>Alternative LIBOR market models</td>
<td>316</td>
</tr>
<tr>
<td>11.5</td>
<td>Swap market models</td>
<td>318</td>
</tr>
<tr>
<td>11.6</td>
<td>Further remarks</td>
<td>321</td>
</tr>
<tr>
<td>11.7</td>
<td>Exercises</td>
<td>321</td>
</tr>
<tr>
<td>12.1</td>
<td>Introduction</td>
<td>323</td>
</tr>
<tr>
<td>12.2</td>
<td>Traditional measures of interest rate risk</td>
<td>323</td>
</tr>
<tr>
<td>12.3</td>
<td>Risk measures in one-factor diffusion models</td>
<td>328</td>
</tr>
<tr>
<td>12.4</td>
<td>Immunization</td>
<td>335</td>
</tr>
<tr>
<td>12.5</td>
<td>Risk measures in multi-factor diffusion models</td>
<td>342</td>
</tr>
<tr>
<td>12.6</td>
<td>Duration-based pricing of options on bonds</td>
<td>346</td>
</tr>
<tr>
<td>12.7</td>
<td>Alternative measures of interest rate risk</td>
<td>351</td>
</tr>
<tr>
<td>12.8</td>
<td>Exercises</td>
<td>354</td>
</tr>
<tr>
<td>13.1</td>
<td>Introduction</td>
<td>355</td>
</tr>
<tr>
<td>13.2</td>
<td>Some basic concepts, relations, and practical issues</td>
<td>357</td>
</tr>
<tr>
<td>13.3</td>
<td>Structural models</td>
<td>368</td>
</tr>
<tr>
<td>13.4</td>
<td>Reduced-form models</td>
<td>384</td>
</tr>
<tr>
<td>13.5</td>
<td>Hybrid models</td>
<td>401</td>
</tr>
<tr>
<td>13.6</td>
<td>Copulas</td>
<td>402</td>
</tr>
<tr>
<td>13.7</td>
<td>Markets for credit derivatives</td>
<td>407</td>
</tr>
<tr>
<td>13.8</td>
<td>Credit default swaps (CDSs)</td>
<td>410</td>
</tr>
<tr>
<td>13.9</td>
<td>Collateralized debt obligations (CDOs)</td>
<td>415</td>
</tr>
</tbody>
</table>
Contents

13.10 Concluding remarks 418
13.11 Exercises 420

14 Mortgages and Mortgage-backed Securities 422
14.1 Introduction 422
14.2 Mortgages 423
14.3 Mortgage-backed bonds 429
14.4 The prepayment option 430
14.5 Rational prepayment models 433
14.6 Empirical prepayment models 445
14.7 Risk measures for mortgage-backed bonds 448
14.8 Other mortgage-backed securities 448
14.9 The subprime crisis 450
14.10 Concluding remarks 452
14.11 Exercises 453

15 Stock and Currency Derivatives When Interest Rates are Stochastic 454
15.1 Introduction 454
15.2 Stock options 454
15.3 Options on forwards and futures 460
15.4 Currency derivatives 464
15.5 Final remarks 470
15.6 Exercises 471

16 Numerical Techniques 472
16.1 Introduction 472
16.2 Numerical solution of PDEs 474
16.3 Monte Carlo simulation 492
16.4 Approximating trees 517
16.5 Concluding remarks 527
16.6 Exercises 528

Appendix A: Results on the Lognormal Distribution 532
References 535
Index 553