Preface xv

About the Authors xix

CHAPTER 1
Introduction 1
  1.1 The Need for Better Financial Modeling of Asset Prices 1
  1.2 The Family of Stable Distribution and Its Properties 5
    1.2.1 Parameterization of the Stable Distribution 5
    1.2.2 Desirable Properties of the Stable Distributions 7
    1.2.3 Considerations in the Use of the Stable Distribution 8
  1.3 Option Pricing with Volatility Clustering 9
    1.3.1 Non-Gaussian GARCH Models 11
  1.4 Model Dependencies 12
  1.5 Monte Carlo 13
  1.6 Organization of the Book 14
  References 15

CHAPTER 2
Probability Distributions 19
  2.1 Basic Concepts 19
  2.2 Discrete Probability Distributions 20
    2.2.1 Bernoulli Distribution 21
    2.2.2 Binomial Distribution 21
    2.2.3 Poisson Distribution 22
  2.3 Continuous Probability Distributions 22
    2.3.1 Probability Distribution Function, Probability Density Function, and Cumulative Distribution Function 23
    2.3.2 Normal Distribution 26
    2.3.3 Exponential Distribution 28
    2.3.4 Gamma Distribution 28
CHAPTER 3

Stable and Tempered Stable Distributions 57

3.1 $\alpha$-Stable Distribution 58
   3.1.1 Definition of an $\alpha$-Stable Random Variable 58
   3.1.2 Useful Properties of an $\alpha$-Stable Random Variable 61
   3.1.3 Smoothly Truncated Stable Distribution 63

3.2 Tempered Stable Distributions 65
   3.2.1 Classical Tempered Stable Distribution 65
   3.2.2 Generalized Classical Tempered Stable Distribution 68
   3.2.3 Modified Tempered Stable Distribution 69
   3.2.4 Normal Tempered Stable Distribution 70
   3.2.5 Kim-Rachev Tempered Stable Distribution 73
   3.2.6 Rapidly Decreasing Tempered Stable Distribution 75

3.3 Infinitely Divisible Distributions 76
   3.3.1 Exponential Moments 80

3.4 Summary 82
CHAPTER 4
Stochastic Processes in Continuous Time

4.1 Some Preliminaries
4.2 Poisson Process
  4.2.1 Compounded Poisson Process
4.3 Pure Jump Process
  4.3.1 Gamma Process
  4.3.2 Inverse Gaussian Process
  4.3.3 Variance Gamma Process
  4.3.4 $\alpha$-Stable Process
  4.3.5 Tempered Stable Process
4.4 Brownian Motion
  4.4.1 Arithmetic Brownian Motion
  4.4.2 Geometric Brownian Motion
4.5 Time-Changed Brownian Motion
  4.5.1 Variance Gamma Process
  4.5.2 Normal Inverse Gaussian Process
  4.5.3 Normal Tempered Stable Process
4.6 Lévy Process
4.7 Summary
References

CHAPTER 5
Conditional Expectation and Change of Measure

5.1 Events, $\sigma$-Fields, and Filtration
5.2 Conditional Expectation
5.3 Change of Measures
  5.3.1 Equivalent Probability Measure
  5.3.2 Change of Measure for Continuous-Time Processes
  5.3.3 Change of Measure in Tempered Stable Processes
5.4 Summary
References

CHAPTER 6
Exponential Lévy Models

6.1 Exponential Lévy Models
8.3 Tempered Stable Processes 193
  8.3.1 Kim-Rachev Tempered Stable Case 196
  8.3.2 Classical Tempered Stable Case 198
8.4 Tempered Infinitely Divisible Processes 199
  8.4.1 Rapidly Decreasing Tempered Stable Case 201
  8.4.2 Modified Tempered Stable Case 202
8.5 Time-Changed Brownian Motion 203
  8.5.1 Classical Tempered Stable Processes 205
  8.5.2 Variance Gamma and Skewed Variance Gamma Processes 206
  8.5.3 Normal Tempered Stable Processes 207
  8.5.4 Normal Inverse Gaussian Processes 208
8.6 Monte Carlo Methods 209
  8.6.1 Variance Reduction Techniques 210
  8.6.2 A Nonparametric Monte Carlo Method 214
  8.6.3 A Monte Carlo Example 216
Appendix 217
References 220

CHAPTER 9
Multi-Tail t-Distribution 225
  9.1 Introduction 225
  9.2 Principal Component Analysis 227
    9.2.1 Principal Component Tail Functions 228
    9.2.2 Density of a Multi-Tail t Random Variable 231
  9.3 Estimating Parameters 232
    9.3.1 Estimation of the Dispersion Matrix 233
    9.3.2 Estimation of the Parameter Set $\Theta$ 233
  9.4 Empirical Results 237
    9.4.1 Comparison to Other Models 237
    9.4.2 Two-Dimensional Analysis 238
    9.4.3 Multi-Tail t Model Check for the DAX 242
  9.5 Summary 244
References 246

CHAPTER 10
Non-Gaussian Portfolio Allocation 247
  10.1 Introduction 247
  10.2 Multifactor Linear Model 248
  10.3 Modeling Dependencies 251
  10.4 Average Value-at-Risk 253
  10.5 Optimal Portfolios 255
CHAPTER 11
Normal GARCH models
11.1 Introduction 271
11.2 GARCH Dynamics with Normal Innovation 272
11.3 Market Estimation 275
11.4 Risk-Neutral Estimation 278
11.4.1 Out-of-Sample Performance 282
11.5 Summary 285
References 285

CHAPTER 12
Smoothly Truncated Stable GARCH Models
12.1 Introduction 287
12.2 A Generalized NGARCH Option Pricing Model 288
12.3 Empirical Analysis 291
12.3.1 Results under the Objective Probability Measure 292
12.3.2 Explaining S&P 500 Option Prices 296
12.4 Summary 306
References 307

CHAPTER 13
Infinitely Divisible GARCH Models
13.1 Stock Price Dynamic 311
13.2 Risk-Neutral Dynamic 312
13.3 Non-Normal Infinitely Divisible GARCH 315
13.3.1 Classical Tempered Stable Model 315
13.3.2 Generalized Tempered Stable Model 317
13.3.3 Kim-Rachev Model 319
13.3.4 Rapidly Decreasing Tempered Stable Model 322
13.3.5 Inverse Gaussian Model 324
13.3.6 Skewed Variance Gamma Model 326
13.3.7 Normal Inverse Gaussian Model 329
13.4 Simulate Infinitely Divisible GARCH 331
Appendix 332
References 334
CHAPTER 14
Option Pricing with Monte Carlo Methods
14.1 Introduction 337
14.2 Data Set 338
  14.2.1 Market Estimation 339
14.3 Performance of Option Pricing Models 346
  14.3.1 In-Sample 346
  14.3.2 Out-of-Sample 352
14.4 Summary 355
References 356

CHAPTER 15
American Option Pricing with Monte Carlo Methods
15.1 American Option Pricing in Discrete Time 358
15.2 The Least Squares Monte Carlo Method 359
15.3 LSM Method in GARCH Option Pricing Model 364
15.4 Empirical Illustration 365
15.5 Summary 372
References 372

Index 373