## Contents

List of Figures .......................... xx
List of Tables ............................ xxiii
Acronyms and abbreviations ............... xxvi

### 1 Concepts, models, and definitions  
1.1 Defining nonlinearity ................. 1
1.2 Where does nonlinearity come from? ... 2
1.3 Stationarity and nonstationarity ...... 3
1.4 Invertibility .......................... 6
1.5 Trends ................................ 7
1.6 Seasonality ............................ 10
1.7 Conditional distributions ............... 10
1.8 Wold’s representation and Volterra expansion 11
1.9 Additive models ......................... 12
1.10 Spectral analysis ....................... 13
1.11 Chaos ................................ 14

### 2 Nonlinear models in economic theory  
2.1 Disequilibrium models ................. 16
2.2 Labour market models .................. 18
   2.2.1 Theory ................................ 18
   2.2.2 Practice ............................. 20
2.3 Exchange rates in a target zone ....... 22
   2.3.1 Theory ................................ 22
   2.3.2 Practice ............................. 24
2.4 Production theory ....................... 25

### 3 Parametric nonlinear models ....... 28
3.1 General considerations ................. 28
3.2 Switching regression models ........... 32
3.2.1 Standard switching regression model  
3.2.2 Vector threshold autoregressive model  
3.3 Markov-switching regression models  
3.4 Smooth transition regression models  
3.4.1 Standard smooth transition regression model  
3.4.2 Additive, multiple, and time-varying STR models  
3.4.3 Vector smooth transition autoregressive model  
3.5 Polynomial models  
3.6 Artificial neural network models  
3.7 Min-max models  
3.8 Nonlinear moving average models  
3.9 Bilinear models  
3.10 Time-varying parameters and state space models  
3.11 Random coefficient and volatility models  

4 The nonparametric approach  
4.1 Introduction  
4.2 Autocovariance and spectrum  
4.3 Density, conditional mean, and conditional variance  
4.3.1 Non-Gaussian marginals  
4.3.2 Conditional quantities  
4.4 Dependence measures for nonlinear processes  
4.4.1 Local measures of dependence  
4.4.2 Global measures of dependence  
4.4.3 Measures based on density and distribution functions  
4.4.4 The copula  

5 Testing linearity against parametric alternatives  
5.1 Introduction  
5.2 Consistent misspecification tests  
5.3 Lagrange multiplier or score test  
5.3.1 Standard case  
5.3.2 Test in stages and a heteroskedasticity-robust version  
5.3.3 Robustifying against conditional heteroskedasticity  
5.4 Locally equivalent alternatives  
5.5 Nonlinear model only identified under the alternative
### Contents

5.5.1 Identification problem 73  
5.5.2 General solution 74  
5.5.3 Lagrange multiplier-type tests 77  
5.5.4 Monte Carlo tests 80  
5.5.5 Giving values to the nuisance parameters 82  
5.6 Testing linearity against unspecified alternatives 83  
5.6.1 Regression Specification Error Test 83  
5.6.2 Tests based on expansions 84  
5.7 Comparing parametric linearity tests using asymptotic relative efficiency 85  
5.7.1 Definition 85  
5.7.2 An example 88  
5.8 Which test to use? 90  

6 Testing parameter constancy 92  
6.1 General considerations 92  
6.2 Generalizing the Chow test 93  
6.2.1 Testing against a single break 93  
6.2.2 Testing against multiple breaks 95  
6.3 Lagrange multiplier type tests 97  
6.3.1 Testing a stationary single-equation model 97  
6.3.2 Testing a stationary vector autoregressive model 100  
6.3.3 Testing a nonstationary vector autoregressive model 102  
6.4 Tests based on recursive estimation of parameters 105  
6.4.1 Cumulative sum tests 105  
6.4.2 Moving sum tests 107  
6.4.3 Fluctuation tests 108  
6.4.4 Tests against stochastic parameters 109  
6.4.5 Testing the constancy of cointegrating relationships 111  

7 Nonparametric specification tests 113  
7.1 Introduction 113  
7.2 Nonparametric linearity tests 114  
7.2.1 Nonparametric tests: the spectral domain 115  
7.2.2 Testing linearity in the conditional mean and conditional variance 116
7.2.3 Estimation 119
7.2.4 Asymptotic theory 120
7.2.5 Finite-sample properties and use of the asymptotics 121
7.2.6 A bootstrap approach to testing 122

7.3 Testing for specific functional forms 123
7.3.1 Tests based on residuals 124
7.3.2 Conditional mean and conditional variance testing 127
7.3.3 Continuous time 129

7.4 Selecting lags 129

7.5 Testing for additivity and interaction 133
7.5.1 Testing in additive models 133
7.5.2 A simulated example 136

7.6 Tests for partial linearity and semiparametric modelling 138

7.7 Tests of independence 140
7.7.1 Traditional tests 140
7.7.2 Rank correlation 141
7.7.3 Frequency based tests 143
7.7.4 BDS test 143
7.7.5 Distribution based tests of independence 145
7.7.6 Generalized spectrum and tests of independence 150
7.7.7 Density based tests of independence 153
7.7.8 Some examples of independence testing 158

8 Models of conditional heteroskedasticity 162
8.1 Autoregressive conditional heteroskedasticity 163
8.1.1 The ARCH model 163

8.2 The Generalized ARCH model 164
8.2.1 Why Generalized ARCH? 164
8.2.2 Families of univariate GARCH models 164
8.2.3 Nonlinear GARCH 167
8.2.4 Time-varying GARCH 169
8.2.5 Moment structure of first-order GARCH models 170
8.2.6 Moment structure of higher-order GARCH models 172
8.2.7 Integrated and fractionally Integrated GARCH 172
8.2.8 Stylized facts and the GARCH model 175
9.6 Estimating parameters
9.6.1 Stationarity 242
9.6.2 Identification 245
9.6.3 Estimation in linear models 245
9.6.4 The nonlinear case 247
9.6.5 Estimation in hidden Markov and mixture models 250

10 Nonparametric models 252
10.1 Additive models 252
10.1.1 Estimation in purely additive models 255
10.1.2 Marginal integration 255
10.1.3 Backfitting and smoothed backfitting 257
10.1.4 Additive models with interactions 260
10.1.5 A simulated example 262
10.1.6 Nonparametric and additive estimation of the conditional variance function 263
10.2 Some related models 269
10.2.1 Functional coefficient autoregressive models 269
10.2.2 Transformation of dependent variables and the ACE algorithm 269
10.2.3 Regression trees, splines, and MARS 270
10.2.4 Quantile regression 270
10.3 Semiparametric models 272
10.3.1 Index models 273
10.3.2 Projection pursuit regression 274
10.3.3 Partially linear models 276
10.4 Robust and adaptive estimation 277

11 Nonlinear and nonstationary models 279
11.1 Long memory models 279
11.2 Linear unit root models 285
11.3 Vector autoregressive processes and linear cointegration 288
11.4 Nonlinear I(1) processes 290
11.5 Nonlinear error correction models 293
11.6 Parametric nonlinear regression 297
11.7 Nonparametric estimation in a nonlinear cointegration type framework 302
11.8 Stochastic unit root models 304

12 Algorithms for estimating parametric nonlinear models 307
12.1 Optimization without derivatives 308
   12.1.1 Grid and line searches 308
   12.1.2 Conjugate directions 309
   12.1.3 Simulated annealing 311
   12.1.4 Evolutionary algorithms 314
12.2 Methods requiring derivatives 317
   12.2.1 Gradient methods 317
   12.2.2 Variable metric methods 322
12.3 Other methods 324
   12.3.1 EM algorithm 324
   12.3.2 Sequential estimation for neural networks 326

13 Basic nonparametric estimates 329
13.1 Density estimation 329
   13.1.1 Kernel estimation 329
   13.1.2 Bias and variance reduction 331
   13.1.3 Choice of bandwidth 333
   13.1.4 Variable bandwidth and nearest neighbour estimation 333
   13.1.5 Multivariate density estimation 334
13.2 Nonparametric regression estimation 334
   13.2.1 Kernel regression estimation 335
   13.2.2 Local polynomial estimation 337
   13.2.3 Bias, convolution, and higher-order kernels 338
   13.2.4 Nearest neighbour estimation 339
   13.2.5 Splines and MARS 341
   13.2.6 Series expansion 341
   13.2.7 Choice of bandwidth for nonparametric regression 342

14 Forecasting from nonlinear models 344
14.1 Introduction 344
14.2 Conditional mean forecasts from parametric models 345
14.2.1 Analytical point forecasts 345
14.2.2 Numerical techniques in forecasting 347
14.3 Forecasting with nonparametric models 351
14.4 Forecast accuracy 354
14.5 The usefulness of forecasts from nonlinear models 356
14.6 Forecasting volatility 361
14.7 Overview of forecasting from nonlinear models 362

15 Nonlinear impulse responses 364
15.1 Generalized impulse response function 364
15.2 Graphical representation 367

16 Building nonlinear models 370
16.1 General considerations 370
16.2 Nonparametric and semiparametric models 371
16.3 Building smooth transition regression models 375
   16.3.1 The three stages of the modelling procedure 375
   16.3.2 Specification 376
   16.3.3 Estimation of parameters 380
   16.3.4 Evaluation 381
   16.3.5 Graphical tools for characterizing the dynamic behaviour of the STAR model 389
   16.3.6 Examples 390
16.4 Building switching regression models 418
   16.4.1 Specification 419
   16.4.2 Estimation and evaluation 422
   16.4.3 Examples 423
16.5 Building artificial neural network models 434
   16.5.1 Specification 435
   16.5.2 Estimation 437
   16.5.3 Evaluation 438
   16.5.4 Alternative modelling approaches 439
   16.5.5 Examples 439
16.6 Two forecast comparisons 445
   16.6.1 Forecasting Wolf’s annual sunspot numbers 445
   16.6.2 Forecasting the monthly US unemployment rate 448