Volume 2: Applications

13 MCMC Methods for Continuous-Time Financial Econometrics 1

Michael Johannes and Nicholas Polson

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**14 The Analysis of the Cross-Section of Security Returns**

Ravi Jagannathan, Georgios Skoulakis, and Zhenyu Wang

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Per A. Mykland

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