CONTENTS

Dedication ................................................... v
Foreword by Edward C. Prescott ......................... vii
Foreword by Harry M. Markowitz ....................... ix
Foreword by James J. Heckman .......................... xi
Foreword by George M. Constantinides ............... xiii
About the Author ........................................... xv

PART I. FINANCIAL MARKETS AND FINANCIAL INSTRUMENTS: BASIC CONCEPTS AND STRATEGIES 1

CHAPTER 1. FINANCIAL MARKETS, FINANCIAL INSTRUMENTS, AND FINANCIAL CRISIS 3

Chapter Outline ............................................ 3
Introduction ................................................... 4
1.1. Trading Characteristics of Commodity Contracts:
   The Case of Oil ........................................... 7
   1.1.1. Fixed prices ........................................ 7
   1.1.2. Floating prices ................................. 8
   1.1.3. Exchange of futures for Physical (EFP) ........ 8
1.2. Description of Markets and Instruments: The Case of the International Petroleum Exchange ................. 8
1.3. Characteristics of Crude Oils and Properties of Petroleum Products .................................... 9
   1.3.1. Specific features of some oil contracts .......... 9
   1.3.2. Description of Markets and Trading Instruments:
   The Brent Market ...................................... 11
1.4. Description of Markets and Trading Instruments: The Case of Cocoa

1.4.1. How do the futures and physicals market work? 14
1.4.2. Arbitrage 14
1.4.3. How is the ICCO price for cocoa beans calculated? 14
1.4.4. Information on how prices are affected by changing economic factors? 15
1.4.5. Cocoa varieties 15
1.4.6. Commodities — Market participants: The case of cocoa, coffee, and white sugar 15

1.5. Trading Characteristics of Options: The Case of Equity Options

1.5.1. Options on equity indices 17
1.5.2. Options on index futures 17
1.5.3. Index options markets around the world 18
1.5.4. Stock Index Markets and the underlying indices in Europe 19

1.6. Trading Characteristics of Options: The Case of Options on Currency Forwards and Futures 21

1.7. Trading Characteristics of Options: The Case of Bonds and Bond Options Markets 22

1.7.1. The specific features of classic interest rate instruments 22
1.7.2. The specific features of mortgage-backed securities 25
1.7.3. The specific features of interest rate futures, options, bond options, and swaps 26

1.8. Simple and Complex Financial Instruments 31

1.9. The Reasons of Financial Innovations 34

1.10. Derivatives Markets in the World: Stock Options, Index Options, Interest Rate and Commodity Options and Futures Markets 36

1.10.1. Global overview 36
1.10.2. The main indexes around the world: a historical perspective 36

Summary 46
Questions 48
Exercises 48
Appendix 56
References 66
CHAPTER 2. RISK MANAGEMENT, DERIVATIVES
MARKETS AND TRADING STRATEGIES

Chapter Outline ................................................. 67
Introduction .................................................. 68

2.1. Introduction to Commodity Markets: The Case of Oil .... 70
   2.1.1. Oil futures markets .................................. 70
   2.1.2. Oil futures exchanges ............................... 70
   2.1.3. Delivery procedures ................................. 70
   2.1.4. The long-term oil market ............................ 71

2.2. Pricing Models ........................................... 71
   2.2.1. The pricing of forward and futures oil contracts ... 71
      2.2.1.1. Relationship to physical market ............... 71
      2.2.1.2. Term structure of prices ..................... 72
   2.2.2. Pricing swaps ....................................... 72
   2.2.3. The pricing of forward and futures commodity contracts: General principles .......... 72
      2.2.3.1. Forward prices and futures prices: Some definitions ............................ 73
      2.2.3.2. Futures contracts on commodities .......... 74
      2.2.3.3. Futures contracts on a security with no income .................. 74
      2.2.3.4. Futures contracts on a security with a known income .................. 74
      2.2.3.5. Futures contracts on foreign currencies ... 75
      2.2.3.6. Futures contracts on a security with a discrete income ............... 75
      2.2.3.7. Valuation of interest rate futures contracts .................. 76
      2.2.3.8. The pricing of future bond contracts ......... 77

2.3. Trading Motives: Hedging, Speculation, and Arbitrage ... 78
   2.3.1. Hedging using futures markets ..................... 78
      2.3.1.1. Hedging: The case of cocoa .................. 79
      2.3.1.2. Hedging: The case of oil .................... 79
      2.3.1.3. Hedging: The case of petroleum products futures contracts ............... 80
      2.3.1.4. The use of futures contracts by petroleum products marketers, jobbers, consumers, and refiners .......... 82
<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.3.2.</td>
<td>Speculation using futures markets</td>
<td>84</td>
</tr>
<tr>
<td>2.3.3.</td>
<td>Arbitrage and spreads in futures markets</td>
<td>84</td>
</tr>
<tr>
<td>2.4.</td>
<td>The Main Bounds on Option Prices</td>
<td>85</td>
</tr>
<tr>
<td>2.4.1.</td>
<td>Boundary conditions for call options</td>
<td>86</td>
</tr>
<tr>
<td>2.4.2.</td>
<td>Boundary conditions for put options</td>
<td>86</td>
</tr>
<tr>
<td>2.4.3.</td>
<td>Some relationships between call options</td>
<td>86</td>
</tr>
<tr>
<td>2.4.4.</td>
<td>Some relationships between put options</td>
<td>88</td>
</tr>
<tr>
<td>2.4.5.</td>
<td>Other properties</td>
<td>89</td>
</tr>
<tr>
<td>2.5.</td>
<td>Simple Trading Strategies for Options and their Underlying Assets</td>
<td>90</td>
</tr>
<tr>
<td>2.5.1.</td>
<td>Trading the underlying assets</td>
<td>90</td>
</tr>
<tr>
<td>2.5.2.</td>
<td>Buying and selling calls</td>
<td>91</td>
</tr>
<tr>
<td>2.5.3.</td>
<td>Buying and selling puts</td>
<td>93</td>
</tr>
<tr>
<td>2.6.</td>
<td>Some Option Combinations</td>
<td>94</td>
</tr>
<tr>
<td>2.6.1.</td>
<td>The straddle</td>
<td>94</td>
</tr>
<tr>
<td>2.6.2.</td>
<td>The strangle</td>
<td>94</td>
</tr>
<tr>
<td>2.7.</td>
<td>Option Spreads</td>
<td>95</td>
</tr>
<tr>
<td>2.7.1.</td>
<td>Bull and bear spreads with call options</td>
<td>95</td>
</tr>
<tr>
<td>2.7.2.</td>
<td>Bull and bear spreads with put options</td>
<td>96</td>
</tr>
<tr>
<td>2.7.3.</td>
<td>Box spread</td>
<td>96</td>
</tr>
<tr>
<td>2.7.3.1.</td>
<td>Definitions and examples</td>
<td>96</td>
</tr>
<tr>
<td>2.7.3.2.</td>
<td>Trading a box spread</td>
<td>98</td>
</tr>
<tr>
<td>2.8.</td>
<td>Butterfly Strategies</td>
<td>99</td>
</tr>
<tr>
<td>2.8.1.</td>
<td>Butterfly spread with calls</td>
<td>99</td>
</tr>
<tr>
<td>2.8.2.</td>
<td>Butterfly spread with puts</td>
<td>100</td>
</tr>
<tr>
<td>2.9.</td>
<td>Condor Strategies</td>
<td>100</td>
</tr>
<tr>
<td>2.9.1.</td>
<td>Condor strategy with calls</td>
<td>100</td>
</tr>
<tr>
<td>2.9.2.</td>
<td>Condor strategy with puts</td>
<td>101</td>
</tr>
<tr>
<td>2.10.</td>
<td>Ratio Spreads</td>
<td>102</td>
</tr>
<tr>
<td>2.11.</td>
<td>Some Combinations of Options with Bonds and Stocks</td>
<td>103</td>
</tr>
<tr>
<td>2.11.1.</td>
<td>Covered call: short a call and hold the underlying asset</td>
<td>103</td>
</tr>
<tr>
<td>2.11.2.</td>
<td>Portfolio insurance</td>
<td>103</td>
</tr>
<tr>
<td>2.11.3.</td>
<td>Mimicking portfolios and synthetic instruments</td>
<td>104</td>
</tr>
<tr>
<td>2.11.3.1.</td>
<td>Mimicking the underlying asset</td>
<td>104</td>
</tr>
<tr>
<td>2.11.3.2.</td>
<td>Synthetic underlying asset: Long call plus</td>
<td>104</td>
</tr>
<tr>
<td></td>
<td>a short put and bonds</td>
<td></td>
</tr>
<tr>
<td>2.11.3.3.</td>
<td>The synthetic put: put-call parity relationship</td>
<td>105</td>
</tr>
</tbody>
</table>
2.12. Conversions and Reversals ................................. 106
2.13. Case study: Selling Calls (Without Holding the Stocks/as an Alternative to Short Selling Stocks/the Idea of Selling Calls is Also an Alternative to Buying Puts) ......................... 107
  2.13.1. Data and assumptions ................................. 107
    2.13.1.1. Selling calls (without holding the stock) 107
    2.13.1.2. Comparing the strategy of selling calls (with a short portfolio of stocks): the extreme case ................................. 109
    2.13.1.3. Selling calls (holding the stock) ................. 110
  2.13.2. Leverage in selling call options (without holding the stocks) ................................................. 110
    2.13.2.1. Selling Call options (without holding the stocks) ................................................. 111
    2.13.2.2. Leverage in selling Call options (without holding the stocks): The extreme case ......... 113
    2.13.2.3. Selling calls using leverage (and holding the stock) ................................................. 113
  2.13.3. Short sale of the stocks without options .......... 113
2.14. Buying Calls on EMA ........................................... 116
  2.14.1. Buying a call as an alternative to buying the stock: (also as an alternative to short sell put options) .......... 116
    2.14.1.1. Data and assumptions ............................ 116
    2.14.1.2. Pattern of risk and return ...................... 116
  2.14.2. Compare buying calls (as an alternative to portfolio of stocks) ................................................. 117
    2.14.2.1. Risk return in options ............................ 118
  2.14.3. Example by changing volatility to 20% ........... 120
    2.14.3.1. Data and assumptions: ............................ 120
    2.14.3.2. Compare buying calls (as an alternative to portfolio of stocks) ................................................. 121
    2.14.3.3. Leverage in buying call options (without selling the underlying) ................................................. 123

Summary .......................................................... 125
Questions .......................................................... 127
Case Study: Comparisons Between put and Call Options ........ 128
1. Buying Puts and Selling Puts Naked .......................... 128
  1.1. Buying puts .................................................. 128
  1.2. Selling puts .................................................. 129
2. Buying and Selling Calls ........................................ 131
   2.1. Buying calls ...................................... 131
   2.2. Selling a call .................................. 132
3. Strategy of Buying a Put and Hedge and Selling a Put and Hedge .............. 132
   3.1. Strategy of selling put and hedge: sell delta units of the underlying .......... 134
   3.2. Strategy of buy put and hedge: buy delta units of the underlying .......... 135
4. Strategy of Buy Call, Sell Put, and Buy Call, Sell Put and Hedge .............. 135
5. Strategy of Buy Call, Sell Put: Equivalent to Holding the Underlying .............. 137
References .......................................................... 140

CHAPTER 3. TRADING OPTIONS AND THEIR UNDERLYING ASSET: RISK MANAGEMENT IN DISCRETE TIME 141
Chapter Outline ...................................................... 141
Introduction .......................................................... 141
3.1. Basic Strategies and Synthetic Positions ................................ 142
   3.1.1. Options and synthetic positions .................................. 142
   3.1.2. Long or short the underlying asset .................................. 144
   3.1.3. Long a call ........................................ 144
   3.1.4. Short call ......................................... 145
   3.1.5. Long a put ......................................... 147
   3.1.6. Short a put ......................................... 148
3.2. Combined Strategies ............................................ 150
   3.2.1. Long a straddle ....................................... 150
   3.2.2. Short a straddle ....................................... 152
   3.2.3. Long a strangle ....................................... 153
   3.2.4. Short a strangle ....................................... 156
   3.2.5. Long a tunnel ........................................ 157
   3.2.6. Short a tunnel ........................................ 158
   3.2.7. Long a call bull spread ................................. 159
   3.2.8. Long a put bull spread ................................. 159
3.2.9. Long a call bear spread ........................................ 161
3.2.10. Selling a put bear spread .................................... 162
3.2.11. Long a butterfly ............................................... 162
3.2.12. Short a butterfly .............................................. 163
3.2.13. Long a condor .................................................. 163
3.2.14. Short a condor .................................................. 164
3.3. How Traders Use Option Pricing Models: Parameter
      Estimation .......................................................... 166
3.3.1. Estimation of model parameters .............................. 167
      3.3.1.1. Historical volatility ...................................... 167
      3.3.1.2. Implied volatilities and option pricing
                  models .................................................... 170
3.3.2. Trading and Greek letters .................................... 170
3.4. Summary ............................................................ 174
Case Studies .................................................................. 176
Exercises ........................................................................ 208
Questions ....................................................................... 175
References ...................................................................... 217

PART II. PRICING DERIVATIVES AND THEIR
UNDERLYING ASSETS IN A DISCRETE-TIME
SETTING ........................................................................ 219

CHAPTER 4. OPTION PRICING: THE DISCRETE-
TIME APPROACH FOR STOCK OPTIONS .......................... 221

Chapter Outline ................................................................ 221
Introduction ........................................................................ 221
4.1. The CRR Model for Equity Options ............................... 222
   4.1.1. The mono-periodic model ...................................... 222
   4.1.2. The multiperiodic model ....................................... 224
   4.1.3. Applications and examples .................................... 226
     4.1.3.1. Applications of the CRR model within
                two periods .................................................. 226
     4.1.3.2. Other applications of the binomial model
                of CRR for two periods .................................... 228
     4.1.3.3. Applications of the binomial model of
                CRR for three periods ..................................... 231
     4.1.3.4. Examples with five periods ............................... 234
4.2. The Binomial Model and the Distributions to the Underlying Assets

4.2.1. The Put-Call parity in the presence of several cash-distributions

4.2.2. Early exercise of American stock options

4.2.3. The model

4.2.4. Simulations for a small number of periods

4.2.5. Simulations in the presence of two dividend dates

4.2.6. Simulations for different periods and several dividends: The general case

Summary

Questions

Appendix: The Lattice Approach

References

CHAPTER 5. CREDIT RISKS, PRICING BONDS, INTEREST RATE INSTRUMENTS, AND THE TERM STRUCTURE OF INTEREST RATES

Chapter Outline

Introduction

5.1. Time Value of Money and the Mathematics of Bonds

5.1.1. Single payment formulas

5.1.2. Uniform-series present worth factor (USPWF) and the capital recovery factor (CRF)

5.1.3. Uniform-series compound-amount factor (USCAF) and the sinking fund factor (SFF)

5.1.4. Nominal interest rates and continuous compounding

5.2. Pricing Bonds

5.2.1. A coupon-paying bond

5.2.2. Zero-coupon bonds

5.3. Computation of the Yield or the Internal Rate of Return

5.3.1. How to measure the yield

5.3.2. The CY

5.3.3. The YTM

5.3.4. The YTC

5.3.5. The potential yield from holding bonds

5.4. Price Volatility Measures: Duration and Convexity

5.4.1. Duration
5.4.2. Duration of a bond portfolio .......................... 273
5.4.3. Modified duration ................................. 273
5.4.4. Price volatility measures: Convexity .............. 274
5.5. The Yield Curve and the Theories of Interest Rates .... 275
5.5.1. The shapes of the yield curve ...................... 276
5.5.2. Theories of the term structure of interest rates .. 276
5.5.2.1. The pure expectations theory .................. 276
5.6. The YTM and the Theories of the Term Structure of Interest Rates .................................. 277
5.6.1. Computing the YTM ............................... 277
5.6.2. Market segmentation theory of the term structure 278
5.7. Spot Rates and Forward Interest Rates .................. 279
5.7.1. The theoretical spot rate ......................... 279
5.7.2. Forward rates ...................................... 279
5.8. Issuing and Redeeming Bonds ........................... 281
5.9. Mortgage-Backed Securities: The Monthly Mortgage Payments for a Level-Payment Fixed-Rate Mortgage .. 283
5.10. Interest Rate Swaps ................................... 286
5.10.1. The pricing of interest rate swaps ............... 286
5.10.2. The swap value as the difference between the prices of two bonds ................................. 286
5.10.3. The valuation of currency swaps .................. 287
5.10.4. Computing the swap .............................. 289
Summary .................................................. 289
Questions .................................................. 290
References ................................................ 291

CHAPTER 6. EXTENSIONS OF SIMPLE BINOMIAL OPTION PRICING MODELS TO INTEREST RATES AND CREDIT RISK 293

Chapter Outline ........................................... 293
Introduction ................................................ 293
6.1. The Rendleman and Bartter Model (for details, refer to Bellalah et al., 1998) for Interest-Rate Sensitive Instruments 294
6.1.1. Using the model for coupon-paying bonds ........ 296
6.2. Ho and Lee Model for Interest Rates and Bond Options 297
6.2.1. The binomial dynamics of the term structure .... 297
6.2.2. The binomial dynamics of bond prices .......... 298
6.2.3. Computation of bond prices in the Ho and Lee model ........................................... 298
6.2.4. Option pricing in the Ho and Lee model ................................................................. 299
6.2.5. Deficiency in the Ho and Lee model ......................................................................... 302

6.3. Binomial Interest-Rate Trees and the Log-Normal Random Walk ........................................... 303
6.4. The Black-Derman-Toy Model (BDT) ........................................................................ 308
  6.4.1. Examples and applications ................................................................................. 309
6.5. Trinomial Interest-Rate Trees and the Pricing of Bonds ................................................. 313
  6.5.1. The model ............................................................................................................. 313
  6.5.2. Applications of the binomial and trinomial models .................................................. 316

Summary .................................................................................................................................. 318
Questions .................................................................................................................................. 320
Appendix A: Ho and Lee model and binomial dynamics of bond prices ................................. 321
References .................................................................................................................................. 325

CHAPTER 7. DERIVATIVES AND PATH-DEPENDENT DERIVATIVES: EXTENSIONS AND GENERALIZATIONS OF THE LATTICE APPROACH BY ACCOUNTING FOR INFORMATION COSTS AND ILLIQUIDITY .................................................................................................................................. 327

Chapter Outline .......................................................................................................................... 327
Introduction .................................................................................................................................. 327

7.1. The Standard Lattice Approach for Equity Options: The Standard Analysis ...................... 329
  7.1.1. The model for options on a spot asset with any pay outs ........................................... 329
  7.1.2. The model for futures options ................................................................................... 330
  7.1.3. The model with dividends ......................................................................................... 330
    7.1.3.1. A known dividend yield ..................................................................................... 331
    7.1.3.2. A known proportional dividend yield .............................................................. 331
    7.1.3.3. A known discrete dividend ................................................................................. 332
  7.1.4. Examples .................................................................................................................... 332
    7.1.4.1. The European put price with dividends ............................................................ 333
    7.1.4.2. The American put price with dividends ............................................................. 333

7.2. A Simple Extension to Account for Information Uncertainty in the Valuation of Futures and Options .......................................................................................................................... 338
7.2.1. On the valuation of derivatives and information costs ......................................................... 338
7.2.2. The valuation of forward and futures contracts in the presence of information costs ........ 340
  7.2.2.1. Forward, futures, and arbitrage ........................................... 340
  7.2.2.2. The valuation of forward contracts in the absence of distributions to the underlying asset ......................................................... 340
  7.2.2.3. The valuation of forward contracts in the presence of a known cash income to the underlying asset ......................................................... 341
  7.2.2.4. The valuation of forward contracts in the presence of a known dividend yield to the underlying asset ......................................................... 341
  7.2.2.5. The valuation of stock index futures ............................................. 342
  7.2.2.6. The valuation of Forward and futures contracts on currencies ........................................... 342
  7.2.2.7. The valuation of futures contracts on silver and gold ........................................... 343
  7.2.2.8. The valuation of Futures on other commodities ........................................... 343
7.2.3. Arbitrage and information costs in the lattice approach ........................................... 343
7.2.4. The binomial model for options in the presence of a continuous dividend stream and information costs ......................................................... 346
7.2.5. The binomial model for options in the presence of a known dividend yield and information costs ......................................................... 347
7.2.6. The binomial model for options in the presence of a discrete dividend stream and information costs ......................................................... 347
7.2.7. The binomial model for futures options in the presence of information costs ......................................................... 347
7.2.8. The lattice approach for American options with information costs and several cash distributions ......................................................... 348
  7.2.8.1. The model ........................................... 348
7.3. The Binomial Model and the Risk Neutrality: Some Important Details ........................................... 349
  7.3.1. The binomial parameters and risk neutrality ........................................... 349
  7.3.2. The convergence argument ........................................... 352
7.4. The Hull and White Trinomial Model for Interest Rate Options .................................................. 353
7.5. Pricing Path-Dependent Interest Rate Contingent Claims Using a Lattice ........................................ 355
  7.5.1. The framework .................................................. 355
  7.5.2. Valuation of the path-dependent security ............ 358
    7.5.2.1. Fixed-coupon rate security ......................... 358
    7.5.2.2. Floating-coupon security ......................... 359
  7.5.3. Options on path-dependent securities ................ 359
    7.5.3.1. Short-dated options ................................. 359
    7.5.3.2. Long-dated options ................................. 359

Summary ................................................................. 360
Questions .............................................................. 362
References ............................................................. 362

PART III. OPTION PRICING IN A CONTINUOUS-TIME SETTING: BASIC MODELS, EXTENSIONS AND APPLICATIONS 365


Chapter Outline ....................................................... 367
Introduction .......................................................... 368
8.1. Precursors to the Black–Scholes Model ..................... 369
  8.1.1. Bachelier formula ....................................... 369
  8.1.2. Sprenkle formula ....................................... 370
  8.1.3. Boness formula ....................................... 371
  8.1.4. Samuelson formula ................................... 371
8.2. How the Black–Scholes Option Formula is Obtained .......... 372
  8.2.1. The short story ....................................... 372
  8.2.2. The differential equation ............................. 373
  8.2.3. The derivation of the formula ......................... 373
  8.2.4. Publication of the formula ............................. 374
  8.2.5. Testing the formula ................................... 374
8.3. Financial Theory and the Black–Scholes–Merton Theory ...... 375
  8.3.1. The Black–Scholes–Merton theory ................. 375
  8.3.2. Analytical formulas .................................. 376
8.4. The Black–Scholes Model .................................................. 377
  8.4.1. The Black–Scholes model and CAPM .......................... 377
  8.4.2. An alternative derivation of the Black–Scholes model ...... 380
  8.4.3. The put-call parity relationship ................................. 382
  8.4.4. Examples ............................................................. 383

8.5. The Black Model for Commodity Contracts ....................... 386
  8.5.1. The model for forward, futures, and option contracts ....... 386
  8.5.2. The put-call relationship ........................................ 388

8.6. Application of the CAPM Model to Forward and Futures Contracts ......................................................... 389
  8.6.1. An application of the model to forward and futures contracts ......................................................... 389
  8.6.2. An application to the derivation of the commodity option valuation ....................................................... 390
  8.6.3. An application to commodity options and commodity futures options ....................................................... 393

8.7. The Holes in the Black–Scholes–Merton Theory and the Financial Crisis ......................................................... 394
  8.7.1. Volatility changes .................................................. 394
  8.7.2. Interest rate changes .............................................. 395
  8.7.3. Borrowing penalties .............................................. 396
  8.7.4. Short-selling penalties ........................................... 396
  8.7.5. Transaction costs .................................................. 396
  8.7.6. Taxes ................................................................. 396
  8.7.7. Dividends ........................................................... 396
  8.7.8. Takeovers ........................................................... 397

Summary .......................................................................... 397
Questions ........................................................................ 398
Appendix A. The Cumulative Normal Distribution Function ..... 399
Appendix B. The Bivariate Normal Density Function ............. 400
References ..................................................................... 401

CHAPTER 9. SIMPLE EXTENSIONS AND APPLICATIONS OF THE BLACK-SCHOLES TYPE MODELS IN VALUATION AND RISK MANAGEMENT 403

Chapter Outline ............................................................. 403
Introduction ..................................................................... 403
9.1. Applications of the Black–Scholes Model...

9.1.1. Valuation and the role of equity options...

9.1.2. Valuation and the role of index options...

9.1.2.1. Analysis and valuation...

9.1.2.2. Arbitrage between index options and futures...

9.1.3. Valuation of options on zero-coupon bonds...

9.1.4. Valuation and the role of short-term options on long-term bonds...

9.1.5. Valuation of interest rate options...

9.1.6. Valuation and the role of bond options: the case of coupon-paying bonds...

9.1.7. The valuation of a swaption...

9.2. Applications of the Black's Model...

9.2.1. Options on equity index futures...

9.2.2. Options on currency forwards and options on currency futures...

9.2.2.1. Options on currency forwards...

9.2.2.2. Options on currency futures...

9.2.3. The Black’s model and valuation of interest rate caps...

9.3. The Extension to Foreign Currencies: The Garman and Kohlhagen Model and its Applications...

9.3.1. The currency call formula...

9.3.2. The currency put formula...

9.3.3. The interest-rate theorem and the pricing of forward currency options...

9.4. The Extension to Other Commodities: The Merton, Barone-Adesi and Whaley Model, and Its Applications...

9.4.1. The model...

9.4.2. An application to portfolio insurance...

9.5. The Real World and the Black–Scholes Type Models...

9.5.1. Volatility...

9.5.2. The hedging strategy...

9.5.3. The log-normal assumption...

9.5.4. A world of finite trading...

9.5.5. Total variance...

9.5.6. Black–Scholes as the limiting case...

9.5.7. Using the model to optimize hedging...
CHAPTER 10. APPLICATIONS OF OPTION PRICING MODELS TO THE MONITORING AND THE MANAGEMENT OF PORTFOLIOS OF DERIVATIVES IN THE REAL WORLD

Chapter Outline

Introduction .................................................................................................................................................. 439

10.1. Option-Price Sensitivities: Some Specific Examples ................................................................................. 441
  10.1.1. Delta .................................................................................................................................................. 441
  10.1.2. Gamma ............................................................................................................................................. 442
  10.1.3. Theta ............................................................................................................................................... 443
  10.1.4. Vega ................................................................................................................................................ 444
  10.1.5. Rho .................................................................................................................................................. 444
  10.1.6. Elasticity ........................................................................................................................................ 445

10.2. Monitoring and Managing an Option Position in Real Time ............................................................... 445
  10.2.1. Simulations and analysis of option price sensitivities using Barone-Adesi and Whaley model .. 446
  10.2.2. Monitoring and adjusting the option position in real time .......................................................... 451
    10.2.2.1. Monitoring and managing the delta .......................................................................................... 451
    10.2.2.2. Monitoring and managing the gamma ..................................................................................... 454
    10.2.2.3. Monitoring and managing the theta ....................................................................................... 457
    10.2.2.4. Monitoring and managing the vega ....................................................................................... 458

10.3. The Characteristics of Volatility Spreads .......................................................................................... 459

Summary ...................................................................................................................................................... 460

Appendix A: Greek-Letter Risk Measures in Analytical Models ................................................................. 461
  A.1. B–S model ........................................................................................................................................... 461
  A.2. Black’s Model ..................................................................................................................................... 462
  A.3. Garman and Kohlhagen’s model ........................................................................................................ 463
  A.4. Merton’s and Barone-Adesi and Whaley’s model ............................................................................. 463

Appendix B: The Relationship Between Hedging Parameters ...................................................................... 464

Appendix C: The Generalized Relationship Between the Hedging Parameters ......................................... 465

Appendix D: A Detailed Derivation of the Greek Letters ............................................................................. 479

Questions .................................................................................................................................................... 478

References ................................................................................................................................................... 489
PART IV. MATHEMATICAL FOUNDATIONS OF OPTION PRICING MODELS IN A CONTINUOUS-TIME SETTING: BASIC CONCEPTS AND EXTENSIONS

CHAPTER 11. THE DYNAMICS OF ASSET PRICES AND THE ROLE OF INFORMATION: ANALYSIS AND APPLICATIONS IN ASSET AND RISK MANAGEMENT

Chapter Outline

Introduction

11.1. Continuous Time Processes for Asset Price Dynamics

11.1.1. Asset price dynamics and Wiener process

11.1.2. Asset price dynamics and the generalized Wiener process

11.1.3. Asset price dynamics and the Ito process

11.1.4. The log-normal property

11.1.5. Distribution of the rate of return

11.2. Ito's Lemma and Its Applications

11.2.1. Intuitive form

11.2.2. Applications to stock prices

11.2.3. Mathematical form

11.2.4. The generalized Ito's formula

11.2.5. Other applications of Ito's formula

11.3. Taylor Series, Ito's Theorem and the Replication Argument

11.3.1. The relationship between Taylor series and Ito's differential

11.3.2. Ito's differential and the replication portfolio

11.3.3. Ito's differential and the arbitrage portfolio

11.3.4. Why are error terms neglected?

11.4. Forward and Backward Equations

11.5. The Main Concepts in Bond Markets and the General Arbitrage Principle

11.5.1. The main concepts in bond pricing

11.5.2. Time-dependent interest rates and information uncertainty

11.5.3. The general arbitrage principle

11.6. Discrete Hedging and Option Pricing

11.6.1. Discrete hedging
Appendix B: Resolution of the Partial Differential Equation for a European Call Option on a Non-Dividend Paying Stock in the Standard Context

Appendix C: Approximation of the Cumulative Normal Distribution

Appendix D: Leibniz's Rule for Integral Differentiation

Appendix E: Pricing Bonds: Mathematical Foundations

Exercises

References

CHAPTER 13. SIMPLE EXTENSIONS AND GENERALIZATIONS OF THE BLACK–SCHOLES TYPE MODELS IN THE PRESENCE OF INFORMATION COSTS

Chapter Outline

Introduction

13.1. Differential Equation for a Derivative Security on a Spot Asset in the Presence of a Continuous Dividend Yield and Information Costs

13.2. The Valuation of Securities Dependent on Several Variables in the Presence of Incomplete Information: A General Method

13.3. The General Differential Equation for the Pricing of Derivatives

13.4. Extension of the Risk-Neutral Argument in the Presence of Information Costs

13.5. Extension to Commodity Futures Prices within Incomplete Information

13.5.1. Differential equation for a derivative security dependent on a futures price in the presence of information costs

13.5.2. Commodity futures prices

13.5.3. Convenience yields

Summary

Questions

Appendix A: A General Equation for Derivative Securities

Appendix B: Extension to the Risk-Neutral Valuation Argument

Exercises

References
PART V. EXTENSIONS OF OPTION PRICING THEORY TO AMERICAN OPTIONS AND INTEREST RATE INSTRUMENTS IN A CONTINUOUS-TIME SETTING: DIVIDENDS, COUPONS AND STOCHASTIC INTEREST RATES

CHAPTER 14. EXTENSION OF ASSET AND RISK MANAGEMENT IN THE PRESENCE OF AMERICAN OPTIONS: DIVIDENDS, EARLY EXERCISE, AND INFORMATION UNCERTAINTY

Chapter Outline
Introduction
14.1. The Valuation of American Options: The General Problem
14.1.1. Early exercise of American calls
14.1.2. Early exercise of American puts
14.1.3. The American put option and its critical stock price
14.2. Valuation of American Commodity Options and Futures Options with Continuous Distributions
14.2.1. Valuation of American commodity options
14.2.2. Examples and applications
14.2.3. Valuation of American futures options
14.2.4. Examples and applications
14.3. Valuation of American Commodity and Futures Options with Continuous Distributions within Information Uncertainty
14.3.1. Commodity option valuation with information costs
14.3.2. Simulation results
14.4. Valuation of American Options with Discrete Cash-Distributions
14.4.1. Early exercise of American options
14.4.2. Valuation of American options with dividends
14.5. Valuation of American Options with Discrete Cash Distributions within Information Uncertainty
14.5.1. The model
14.5.2. Simulation results
14.6. The Valuation Equations for Standard and Compound Options with Information Costs
14.6.1. The pricing of assets under incomplete information
14.6.2. The valuation of equity as a compound option
CHAPTER 15. RISK MANAGEMENT OF BONDS AND INTEREST RATE SENSITIVE INSTRUMENTS IN THE PRESENCE OF STOCHASTIC INTEREST RATES AND INFORMATION UNCERTAINTY: THEORY AND TESTS

Chapter Outline

15.1. The Valuation of Bond Options and Interest Rate Options

15.1.1. The problems in using the B–S model for interest-rate options

15.1.2. Sensitivity of the theoretical option prices to changes in factors

15.2. A Simple Non-Parametric Approach to Bond Futures Option Pricing

15.2.1. Canonical modeling and option pricing theory

15.2.2. Assessing the distribution of the underlying futures price

15.2.3. Transforming actual probabilities into risk-neutral probabilities

15.2.4. Qualitative comparison of Black and canonical model values

15.3. One-Factor Interest Rate Modeling and the Pricing of Bonds: The General Case

15.3.1. Bond pricing in the general case: The arbitrage argument and information costs

15.3.2. Pricing callable bonds within information uncertainty

15.4. Fixed Income Instruments as a Weighted Portfolio of Power Options

15.5. Merton’s Model for Equity Options in the Presence of Stochastic Interest Rates: Two-Factor Models
15.5.1. The model in the presence of stochastic interest rates ........................................ 679
15.5.2. Applications of Merton's model ................................................................. 680
15.6. Some Models for the Pricing of Bond Options .............................................. 681
15.6.1. An extension of the Ho-Lee model for bond options ................................. 681
15.6.2. The Schaefer and Schwartz model .............................................................. 683
15.6.3. The Vasicek (1977) model ...................................................................... 683
15.6.4. The Ho and Lee model ............................................................................. 684
15.6.5. The Hull and White model ...................................................................... 685
Summary ............................................................................................................ 686
Questions ............................................................................................................ 687
Appendix A: Government Bond Futures and Implicit Embedded Options ......................... 687
   A.1. Criteria for the CTD ................................................................................ 688
   A.2. Yield changes .......................................................................................... 688
   A.3. The value for a futures position ............................................................. 690
   A.4. Parallel yield shift .................................................................................. 691
   A.5. Relative yield shift ................................................................................ 692
Appendix B: One-Factor Fallacies for Interest Rate Models .................................................. 692
   B.1. The models in practice ............................................................................ 693
   B.2. Spreads between rates ............................................................................ 693
Appendix C: Merton's Model in the Presence of Stochastic Interest Rates ......................... 694
References ......................................................................................................... 701

CHAPTER 16. MODELS OF INTEREST RATES, INTEREST-RATE SENSITIVE INSTRUMENTS, AND THE PRICING OF BONDS: THEORY AND TESTS 703

Chapter Outline .................................................................................................... 703
Introduction ........................................................................................................ 704
16.1. Interest Rates and Interest-Rate Sensitive Instruments ........................................... 705
   16.1.1. Zero-coupon bonds .............................................................................. 705
   16.1.2. Term structure of interest rates ........................................................... 705
   16.1.3. Forward interest rates .......................................................................... 706
   16.1.4. Short-term interest rate ........................................................................ 707
   16.1.5. Coupon-bearing bonds ......................................................................... 707
   16.1.6. Yield-to-Maturity (YTM) ..................................................................... 708
   16.1.7. Market conventions ............................................................................. 709
16.2. Interest Rates and the Pricing of Bonds .......................... 710
  16.2.1. The instantaneous interest rates under certainty ............ 710
  16.2.2. The instantaneous interest rate under uncertainty ......... 711
16.3. Interest Rate Processes and the Pricing of Bonds and Options 712
  16.3.1. The Vasicek model ........................................... 713
  16.3.2. The Brennan and Schwartz model .............................. 713
  16.3.3. The CIR model ................................................ 713
  16.3.4. The Ho and Lee model ....................................... 714
  16.3.5. The HJM model ................................................. 714
  16.3.6. The BDT model ............................................... 716
  16.3.7. The Hull and White model ................................... 717
  16.3.8. Fong and Vasicek model ..................................... 718
  16.3.9. Longstaff and Schwartz model ................................ 718
16.4. The Relative Merits of the Competing Models ..................... 718
16.5. A Comparative Analysis of Term Structure Estimation Models .... 721
  16.5.1. The construction of the term structure and coupon bonds ... 721
  16.5.2. Fitting functions and estimation procedure .................. 722
16.7. Distributional Properties of Spot and Forward Interest Rates: USD, DEM, GBP, and JPY ................................. 726
  16.7.1. Interest rate levels .......................................... 728
  16.7.2. Interest rate differences and log differences ............. 728
Summary ........................................................................ 731
Appendix A: An Application of Interest Rate Models to Account for Information Costs: An Exercise ................. 732
  A.1. An application of the HJM model in the presence of information costs ............................................. 732
    A.1.1. The forward rate equation .................................... 732
    A.1.2. The spot rate process ......................................... 733
    A.1.3. The market price of risk ...................................... 734
    A.1.4. Relationship between risk-neutral forward rate drift and volatility .......................................... 735
    A.1.5. Pricing derivatives ............................................ 735
  A.2. An application of the Ho and Lee model in the presence of information cost ......................................... 736
Appendix B: Implementation of the BDT Model with Different Volatility Estimators

B.1. The BDT model ........................................ 737
B.2. Estimation results .................................... 738

Questions ................................................ 739
References ................................................ 740

PART VI. GENERALIZATION OF OPTION PRICING MODELS AND STOCHASTIC VOLATILITY 743

CHAPTER 17. EXTREME MARKET MOVEMENTS, RISK AND ASSET MANAGEMENT: GENERALIZATION TO JUMP PROCESSES, STOCHASTIC VOLATILITIES, AND INFORMATION COSTS 745

Chapter Outline .......................................... 745
Introduction ............................................ 745
17.1. The Jump-Diffusion and the Constant Elasticity of Variance Models ................................ 747
17.1.1. The jump-diffusion model ....................... 747
17.1.2. The constant elasticity of variance diffusion (CEV) process ................................ 748
17.2. On Jumps, Hedging and Information Costs ............... 749
17.2.1. Hedging in the presence of jumps ............... 750
17.2.2. Hedging the jumps ................................ 752
17.2.3. Jump volatility .................................. 753
17.3. On the Smile Effect and Market Imperfections in the Presence of Jumps and Incomplete Information .......... 754
17.3.1. On smiles and jumps ........................... 754
17.3.2. On smiles, jumps, and incomplete information .. 759
17.3.3. Empirical results in the presence of jumps and incomplete information .............. 760
17.4. Implied Volatility and Option Pricing Models: The Model and Simulation Results .................. 763
17.4.1. The valuation model ............................ 763
17.4.2. Simulation results ............................. 765
17.4.3. Model calibration and the smile effect ........... 766
Summary .................................................. 767
Questions ................................................ 768
References ................................................ 768
CHAPTER 18. RISK MANAGEMENT DURING ABNORMAL MARKET CONDITIONS: FURTHER GENERALIZATION TO JUMP PROCESSES, STOCHASTIC VOLATILITIES, AND INFORMATION COSTS

Chapter Outline .................................................. 771
Introduction .......................................................... 772

18.1. Option Pricing in the Presence of a Stochastic Volatility ............................................. 774
   18.1.1. The Hull and White model ................. 774
   18.1.2. Stein and Stein model ...................... 775
   18.1.3. The Heston model ......................... 777
   18.1.4. The Hoffman, Platen, and Schweizer model 778
   18.1.5. Market price of volatility risk ........... 781
   18.1.6. The market price of risk for traded assets 782

18.2. Generalization of Some Models with Stochastic Volatility and Information Costs .................. 782
   18.2.1. Generalization of the Hull and White (1987) model 782
   18.2.2. Generalization of Wiggins's model .......... 784
   18.2.3. Generalization of Stein and Stein's model 785
   18.2.4. Generalization of Heston's model .......... 786
   18.2.5. Generalization of Johnson and Shanno's model 788

18.3. The Volatility Smiles: Some Standard Results .............................................................. 788
   18.3.1. The smile effect in stock options and index options 788
   18.3.2. The smile effect for bond and currency options . 789
   18.3.3. Volatility smiles: Empirical evidence ........ 790

18.4. Empirical Results Regarding Information Costs and Option Pricing .................................. 791
   18.4.1. Information costs and option pricing:
      The estimation method ......................... 791
   18.4.2. The asymmetric distortion of the smile .... 792
   18.4.3. Asymmetric Smiles and information costs in
      a stochastic volatility model ................. 793

Summary ............................................................. 795
Questions ........................................................... 796
References .......................................................... 796
PART VII. OPTION PRICING MODELS AND NUMERICAL ANALYSIS 799

CHAPTER 19. RISK MANAGEMENT, NUMERICAL METHODS AND OPTION PRICING 801

Chapter Outline .................................................. 801
Introduction ...................................................... 801
   19.1.1. The implicit difference scheme 804
   19.1.2. Explicit difference scheme 806
   19.1.3. An extension to account for information costs 807
19.2. Application to European Options on Non-Dividend Paying Stocks 807
   19.2.1. The analytic solution 808
   19.2.2. The numerical solution 808
   19.2.3. An application to European calls on non-dividend paying stocks in the presence of information costs 810
19.3. Valuation of American Options with a Composite Volatility 811
   19.3.1. The effect of interest rate volatility on the index volatility 811
   19.3.2. Valuation of index options with a composite volatility 812
   19.3.3. Numerical solutions and simulations 813
19.4. Simulation Methods: Monte–Carlo Method 817
   19.4.1. Simulation of Gaussian variables 818
   19.4.2. Relationship between option values and simulation methods 818
Summary .......................................................... 819
Questions .......................................................... 820
Appendix A: Simple Concepts in Numerical Analysis 820
Appendix B: An Algorithm for a European Call 822
Appendix C: The Algorithm for the Valuation of American Long-term Index Options with a Composite Volatility 823
CHAPTER 20. NUMERICAL METHODS AND PARTIAL DIFFERENTIAL EQUATIONS FOR EUROPEAN AND AMERICAN DERIVATIVES WITH COMPLETE AND INCOMPLETE INFORMATION

Chapter Outline

Introduction................................................................................. 834
20.1. Valuation of American Calls on Dividend-Paying Stocks .... 835
   20.1.1. The Schwartz model .................................................... 835
   20.1.2. The numerical solution ............................................... 836
20.2. American Puts on Dividend-Paying Stocks .................... 837
   20.2.1. The Brennan and Schwartz model ............................... 837
   20.2.2. The numerical solution ............................................... 838
   20.3.1. Finite difference methods in the presence of information costs ................................................................. 839
   20.3.2. An application to the American put using explicit or implicit finite difference methods ................................. 841
20.4. Convertible Bonds .............................................................. 842
   20.4.1. Specific features of CB .................................................. 842
   20.4.2. The valuation equations ............................................... 843
   20.4.3. The numerical solution ............................................... 845
   20.4.4. Simulations ................................................................. 847
20.5. Two-Factor Interest Rate Models and Bond Pricing within Information Uncertainty .............................................. 847
20.6. CBs Pricing within Information Uncertainty .................... 850
   20.6.1. The pricing of CBs ....................................................... 850
   20.6.2. Specific call and put features ....................................... 851
   20.6.3. The pricing of CBs in two-factor models within information uncertainty ............................................................ 851
Summary................................................................................... 852
Appendix A: A Discretizing Strategy for Mean-Reverting Models . 853
Appendix B: An Algorithm for the American Call with Dividends 861
APPENDIX C: THE ALGORITHM FOR THE AMERICAN PUT WITH DIVIDENDS

APPENDIX D: THE ALGORITHM FOR CBs WITH CALL AND PUT PRICES

QUESTIONS

EXERCISES

REFERENCES

PART VIII. EXOTIC DERIVATIVES

CHAPTER 21. RISK MANAGEMENT: EXOTICS AND SECOND-GENERATION OPTIONS

Chapter Outline

Introduction

21.1. Exchange Options

21.2. Forward-Start Options

21.3. Pay-Later Options

21.4. Simple Chooser Options

21.5. Complex Choosers

21.6. Compound Options
  21.6.1. The call on a call in the presence of a cost of carry
  21.6.2. The put on a call in the presence of a cost of carry
  21.6.3. The formula for a call on a put in the presence of a cost of carry
  21.6.4. The put on a put in the presence of a cost of carry

21.7. Options on the Maximum (Minimum)
  21.7.1. The call on the minimum of two assets
  21.7.2. The call on the maximum of two assets
  21.7.3. The put on the minimum (maximum) of two assets

21.8. Extendible Options
  21.8.1. The valuation context
  21.8.2. Extendible calls

21.9. Equity-Linked Foreign Exchange Options and Quantos
  21.9.1. The foreign equity call struck in foreign currency
  21.9.2. The foreign equity call struck in domestic currency
  21.9.3. Fixed exchange rate foreign equity call
  21.9.4. An equity-linked foreign exchange call

21.10. Binary Barrier Options
  21.10.1. Path-independent binary options
21.10.1.1. Standard cash-or-nothing options ... 902
21.10.1.2. Cash-or-nothing options with shadow costs ... 903
21.10.1.3. Standard asset-or-nothing options ... 904
21.10.1.4. Asset-or-nothing options with shadow costs ... 905
21.10.1.5. Standard gap options ... 906
21.10.1.6. Gap options with shadow costs ... 908
21.10.1.7. Supershares ... 908

21.11. Lookback Options ... 908
21.11.1. Standard lookback options ... 909
21.11.2. Options on extrema ... 909
21.11.2.1. On the maximum ... 909
21.11.2.2. On the minimum ... 910
21.11.3. Limited risk options ... 910
21.11.4. Partial lookback options ... 911

Summary ... 913
Questions ... 914
References ... 914

CHAPTER 22. VALUE AT RISK, CREDIT RISK, AND CREDIT DERIVATIVES 917

Chapter Outline ... 917
Introduction ... 917

22.1. VaR and Riskmetrics: Definitions and Basic Concepts ... 919
22.1.1. The definition of risk ... 920
22.1.2. VaR: Definition ... 920

22.2. Statistical and Probability Foundation of VaR ... 921
22.2.1. Using percentiles or quantiles to measure market risk ... 922
22.2.2. The choice of the horizon ... 922

22.3. A More Advanced Approach to VaR ... 923

22.4. Credit Valuation and the Creditmetrics Approach ... 927
22.4.1. The portfolio context of credit ... 927
22.4.2. Different credit risk measures ... 927
22.4.3. Stand alone or single exposure risk calculation ... 928
22.4.4. Differing exposure type ... 928