

Rob Kaas • Marc Goovaerts
Jan Dhaene • Michel Denuit

Modern Actuarial Risk Theory

Using **R**

Second Edition

 Springer

Contents



*There are 10^{11} stars in the galaxy. That used to be a huge number. But it's only a hundred billion. It's less than the national deficit! We used to call them astronomical numbers. Now we should call them economical numbers —
Richard Feynman (1918–1988)*

1	Utility theory and insurance	1
1.1	Introduction	1
1.2	The expected utility model	2
1.3	Classes of utility functions	5
1.4	Stop-loss reinsurance	8
1.5	Exercises	13
2	The individual risk model	17
2.1	Introduction	17
2.2	Mixed distributions and risks	18
2.3	Convolution	25
2.4	Transforms	28
2.5	Approximations	30
2.5.1	Normal approximation	30
2.5.2	Translated gamma approximation	32
2.5.3	NP approximation	33
2.6	Application: optimal reinsurance	35
2.7	Exercises	36
3	Collective risk models	41
3.1	Introduction	41
3.2	Compound distributions	42
3.2.1	Convolution formula for a compound cdf	44
3.3	Distributions for the number of claims	45
3.4	Properties of compound Poisson distributions	47
3.5	Panjer's recursion	49
3.6	Compound distributions and the Fast Fourier Transform	54
3.7	Approximations for compound distributions	57
3.8	Individual and collective risk model	59
3.9	Loss distributions: properties, estimation, sampling	61
3.9.1	Techniques to generate pseudo-random samples	62
3.9.2	Techniques to compute ML-estimates	63

3.9.3	Poisson claim number distribution	63
3.9.4	Negative binomial claim number distribution	64
3.9.5	Gamma claim severity distributions	66
3.9.6	Inverse Gaussian claim severity distributions	67
3.9.7	Mixtures/combinations of exponential distributions	69
3.9.8	Lognormal claim severities	71
3.9.9	Pareto claim severities	72
3.10	Stop-loss insurance and approximations	73
3.10.1	Comparing stop-loss premiums in case of unequal variances	76
3.11	Exercises	78
4	Ruin theory	87
4.1	Introduction	87
4.2	The classical ruin process	89
4.3	Some simple results on ruin probabilities	91
4.4	Ruin probability and capital at ruin	95
4.5	Discrete time model	98
4.6	Reinsurance and ruin probabilities	99
4.7	Beekman's convolution formula	101
4.8	Explicit expressions for ruin probabilities	106
4.9	Approximation of ruin probabilities	108
4.10	Exercises	111
5	Premium principles and Risk measures	115
5.1	Introduction	115
5.2	Premium calculation from top-down	116
5.3	Various premium principles and their properties	119
5.3.1	Properties of premium principles	120
5.4	Characterizations of premium principles	122
5.5	Premium reduction by coinsurance	125
5.6	Value-at-Risk and related risk measures	126
5.7	Exercises	133
6	Bonus-malus systems	135
6.1	Introduction	135
6.2	A generic bonus-malus system	136
6.3	Markov analysis	138
6.3.1	Loimaranta efficiency	141
6.4	Finding steady state premiums and Loimaranta efficiency	142
6.5	Exercises	146
7	Ordering of risks	149
7.1	Introduction	149
7.2	Larger risks	152
7.3	More dangerous risks	154
7.3.1	Thicker-tailed risks	154

7.3.2	Stop-loss order	159
7.3.3	Exponential order	160
7.3.4	Properties of stop-loss order	160
7.4	Applications	164
7.4.1	Individual versus collective model	164
7.4.2	Ruin probabilities and adjustment coefficients	164
7.4.3	Order in two-parameter families of distributions	166
7.4.4	Optimal reinsurance	168
7.4.5	Premiums principles respecting order	169
7.4.6	Mixtures of Poisson distributions	169
7.4.7	Spreading of risks	170
7.4.8	Transforming several identical risks	170
7.5	Incomplete information	171
7.6	Comonotonic random variables	176
7.7	Stochastic bounds on sums of dependent risks	183
7.7.1	Sharper upper and lower bounds derived from a surrogate	183
7.7.2	Simulating stochastic bounds for sums of lognormal risks	186
7.8	More related joint distributions; copulas	190
7.8.1	More related distributions; association measures	190
7.8.2	Copulas	194
7.9	Exercises	196
8	Credibility theory	203
8.1	Introduction	203
8.2	The balanced Bühlmann model	204
8.3	More general credibility models	211
8.4	The Bühlmann–Straub model	214
8.4.1	Parameter estimation in the Bühlmann–Straub model	217
8.5	Negative binomial model for the number of car insurance claims	222
8.6	Exercises	227
9	Generalized linear models	231
9.1	Introduction	231
9.2	Generalized Linear Models	234
9.3	Some traditional estimation procedures and GLMs	237
9.4	Deviance and scaled deviance	245
9.5	Case study I: Analyzing a simple automobile portfolio	248
9.6	Case study II: Analyzing a bonus-malus system using GLM	252
9.6.1	GLM analysis for the total claims per policy	257
9.7	Exercises	262
10	IBNR techniques	265
10.1	Introduction	265
10.2	Two time-honored IBNR methods	268
10.2.1	Chain ladder	268

- 10.2.2 Bornhuetter-Ferguson 270
- 10.3 A GLM that encompasses various IBNR methods 271
 - 10.3.1 Chain ladder method as a GLM 272
 - 10.3.2 Arithmetic and geometric separation methods 273
 - 10.3.3 De Vijlder’s least squares method 274
- 10.4 Illustration of some IBNR methods 276
 - 10.4.1 Modeling the claim numbers in Table 10.1 277
 - 10.4.2 Modeling claim sizes 279
- 10.5 Solving IBNR problems by R 281
- 10.6 Variability of the IBNR estimate 283
 - 10.6.1 Bootstrapping 285
 - 10.6.2 Analytical estimate of the prediction error 288
- 10.7 An IBNR-problem with known exposures 290
- 10.8 Exercises 292

- 11 More on GLMs 297**
 - 11.1 Introduction 297
 - 11.2 Linear Models and Generalized Linear Models 297
 - 11.3 The Exponential Dispersion Family 300
 - 11.4 Fitting criteria 305
 - 11.4.1 Residuals 305
 - 11.4.2 Quasi-likelihood and quasi-deviance 306
 - 11.4.3 Extended quasi-likelihood 308
 - 11.5 The canonical link 310
 - 11.6 The IRLS algorithm of Nelder and Wedderburn 312
 - 11.6.1 Theoretical description 313
 - 11.6.2 Step-by-step implementation 315
 - 11.7 Tweedie’s Compound Poisson–gamma distributions 317
 - 11.7.1 Application to an IBNR problem 318
 - 11.8 Exercises 320

- The ‘R’ in Modern ART 325**
 - A.1 A short introduction to R 325
 - A.2 Analyzing a stock portfolio using R 332
 - A.3 Generating a pseudo-random insurance portfolio 338

- Hints for the exercises 341**

- Notes and references 357**

- Tables 367**

- Index 371**